Lecture 1: Introduction & Preliminaries

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Statistics (MAT1003)

April 10, 2012

Outline

1. **Motivation**
   - What is probability theory and what is statistics?
   - Why studying probability theory and statistics?
   - What will we achieve in this course?
2. **Practicalities**
   - Getting to know each other
   - The rules
3. **Preliminaries: Combinatorics**
   - Permutations
   - Variations & combinations
   - Exercises
   - Homework - recommended exercises
   - Homework - bonus exercises

And now . . .

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Probability theory

Probability theory is a branch of mathematics that deals with calculating the likelihood of an occurrence of a given event.

Statistics

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Motivation: Dependence on uncertainty & biased measurements

- World more and more quantitative and data focused
- Decisions made in the face of uncertainty, with biased measurements
- Quantitative abilities and statistical knowledge needed
- Examples:

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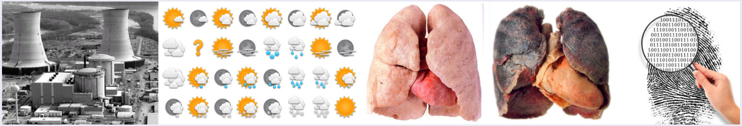

What will we achieve in this course?

Goals of this course

1. To have an understanding of fundamental concepts in probability and statistics
2. To be familiar with the most frequently used probability distributions/densities and statistical procedures (statistical estimation and hypothesis tests)
3. To be able to recognize several probability distributions in real life situations to which they typically apply

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Getting to know each other

Tutors

- Kateřina Staňková
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  www.stankova.net
- Teresa Piovesan
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Rules of the game . . .

- Exam: 0–100 points
- Homework:
  - 0–10 bonus points (average of all homeworks)
  - time to deliver homework: 1 week
  - deadline for today homework: April 16
- Materials:
  - lectures (slides to appear on EleUM and on http://stankova.net/statistics_2012.htm)
  - the book: http://stankova.net/book.pdf – if you follow lectures, you will use it mainly for exercises
- Knowledge I expect you to have: Integration

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About this topic ...

- "Warming up" ...
- Literature: Section 2.3 (although we approach it slightly differently)
- "Counting" the number (♯) of possibilities to select element(s) from a group of element(s)
- Necessary step in learning how to compute probability of random events...

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- Necessary step in learning how to compute probability of random events. . .

Do you know it all?

Book (pp. 51-52): Exercises 2.29, 2.31, 2.33, 2.35, 2.37, 2.39, 2.41, 2.45, 2.47

First Example

Order does not matter

In how many ways can we select

- 1 piece of fruit?
- 1 apple?
- 2 pears?

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Second Example

The same questions, but now order matters

First Example ("A_1 P_1 A_2 P_2 A_3 A_4 P_3 A_5 P_4")

How many ways to select

- 1 piece of fruit:
- 1 apple:
- 2 pears:

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How many ways to select

- 1 piece of fruit:
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How many ways to select

- 1 piece of fruit: 9 ways
- 1 apple:
- 2 pears:

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First Example ("A_1 P_1 A_2 P_2 A_3 A_4 P_3 A_5 P_4")

How many ways to select

- 1 piece of fruit: 9 ways
- 1 apple: 5 ways
- 2 pears:

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- 1 apple: 5 ways
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First Example ("A_1 P_1 A_2 P_2 A_3 A_4 P_3 A_5 P_4")

How many ways to select

- 1 piece of fruit: 9 ways
- 1 apple: 5 ways
- 2 pears: $\frac{4 \cdot 3}{2} = 6$ ways
  $\{P_1\ P_2\}, \{P_1\ P_3\}, \{P_1\ P_4\}, \{P_2\ P_3\}, \{P_2\ P_4\}, \{P_3\ P_4\}$

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Order does not matter (unordered selection, combination)$\Rightarrow$ we select groups rather than individual positions of fruit

Second Example ("A_1 P_1 A_2 P_2 A_3 A_4 P_3 A_5 P_4")

How many ways to select

- 1 piece of fruit:
- 1 apple:
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Second Example ("$A_1\ P_1\ A_2\ P_2\ A_3\ A_4\ P_3\ A_5\ P_4$")

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How many ways to select

- 1 piece of fruit: 9 ways
- 1 apple: 5 ways
- 2 pears: $4 \cdot 3 = 12$ ways

  $(P_1\ P_2)$, $(P_1\ P_3)$, $(P_1\ P_4)$, $(P_2\ P_3)$, $(P_2\ P_4)$, $(P_3\ P_4)$ and the reversed pairs

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Order does matter (ordered selection, variation) $\Rightarrow$ we select individual positions of fruit rather than the groups

Same examples ("A_1 P_1 A_2 P_2 A_3 A_4 P_3 A_5 P_4")

How do you select 3 apples if order matters and if order does not matter?

- Order matters:
- Order does not matter:

Ordered vs. unordered selection

Suppose we select $\{A_1\ A_2\ A_3\}$

- Unordered selection (combination)
- Ordered selection (variation)

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How do you select 3 apples if order matters and if order does not matter?

- Order matters: $5 \cdot 4 \cdot 3 = 60$ ways
- Order does not matter:

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Suppose we select $\{A_1\ A_2\ A_3\}$

- Unordered selection (combination): 1 way
- Ordered selection (variation):

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This holds for any 3 selected apples
$\Rightarrow \sharp$ ordered solutions $= 6 \cdot \sharp$ unordered solutions

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This holds for any 3 selected apples
$\Rightarrow \sharp$ ordered solutions $= 6 \cdot \sharp$ unordered solutions
6 is $\sharp$ orderings of a group of size 3

What is a permutation?

Permutation is an ordering of all elements of a group (book slightly different, Def. 2.7)

♯ permutations of groups of different sizes

- size 1: 1 permutation
- size 2: $2 \cdot 1 = 2$ permutations
- size 3: $3 \cdot 2 \cdot 1 = 6$ permutations
- size 4: $4 \cdot 3 \cdot 2 \cdot 1 = 24$ permutations

  $\vdots$
- size n : $n \cdot (n-1) \cdot \ldots 3 \cdot 2 \cdot 1 = n!$ (n-factorial)

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$\sharp$ permutations of groups of different sizes

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- size 2: $2 \cdot 1 = 2$ permutations
- size 3: $3 \cdot 2 \cdot 1 = 6$ permutations
- size 4: $4 \cdot 3 \cdot 2 \cdot 1 = 24$ permutations

  $\vdots$
- size n : $n \cdot (n-1) \cdot \ldots 3 \cdot 2 \cdot 1 = n!$ (n-factorial)

Back to example ("A_1 P_1 A_2 P_2 A_3 A_4 P_3 A_5 P_4")

How many ways to pick 4 pieces of fruit?

- Ordered – variations:
- Unordered – combinations:
- Can we write these numbers in terms of permutations?

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How many ways to pick 4 pieces of fruit?

- Ordered – variations: $9 \cdot 8 \cdot 7 \cdot 6 = 3024$
  - 9 for the first piece
  - 8 for the second piece
  - 7 for the third piece
  - 6 for the fourth piece
- Unordered – combinations:
- Can we write these numbers in terms of permutations?

Back to example ("$A_1\ P_1\ A_2\ P_2\ A_3\ A_4\ P_3\ A_5\ P_4$")

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- Can we write these numbers in terms of permutations?
  - Ordered - variations: $9 \cdot 8 \cdot 7 \cdot 6 = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{9!}{5!}$
  - Unordered - combinations: $\frac{9!}{5!4!} = \binom{9}{5} = \binom{9}{4}$

    ("combination/variation of 9 (elements) choose 5")

Variation

- Take a set S of n different elements. Choose k elements in a specific order. Each such choice is called a variation of n elements choose k
- $\sharp$ variations: $\frac{n!}{(n-k)!}$

(book: “permutation” Thm 2.4)

Combination

- Take a set S of n different elements. Choose set of k elements, in an unordered manner. Each such choice is called a combination of n elements choose k
- $\sharp$ combinations: $\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n!}{(n-k)!k!} = \binom{n}{n-k}$

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1. How many ways are there to pick 2 letters out of 3 A's and 4 B's? (both ordered and unordered)
2. ...pick 2 A's (both ordered and unordered)?
3. ...pick 2 B's (both ordered and unordered)?
4. ...pick 2 A's and 2 B's (both ordered and unordered)?
5. How many ways to put 2 persons on 5 available seats?
6. How many ways to pick 2 chairs out of 5?
7. How many ways to throw a die and pick a card out of a deck (52 cards?) (Thm. 2.1)
8. How many ways to throw an odd number and pick a spade (♠) or to throw a 2 and pick a 3?
9. How many ways to throw an odd number and pick a spade (♠) or to throw a 3 and pick a 3?
10. How many ways are there to pick 2 A's and 2 B's out of A A's and 4 B's, when we distinguish between B's, but not between A's ("semi-ordered")?

Check yourself

- Book (pp. 51-52): Exercises 2.29, 2.31, 2.33, 2.35, 2.37, 2.39, 2.41, 2.45, 2.47

Notation

$A, B \ldots \Rightarrow$ unordered

$A_1, A_1, B_1, \ldots \Rightarrow$ ordered

Homework - bonus exercises

Bring solutions to the lecture on April 16

1. How many ways to order letters (a) ABCDEF, (b) AABCDE, (c) AABBBC, (d) MAASTRICHT, (e) MISSISSIPPI? (Thms 2.6 and 2.7)
2. How many ways to divide 7 apples among 4 kids (both ordered and unordered)?
3. Book (pp. 51-52): Exercises 2.32, 2.42
4. Book (pp. 42-43): 2.16, 2.58