Lecture 16: Hypothesis Testing

Kateřina Staňková

Statistics (MAT1003)

May 21, 2012

Outline

book: Chapter 10

And now . . .

1. **Revision: Estimation Problems**
2. Revision: Central Limit Theorem
3. Hypothesis testing
   - Basics
   - Errors
   - Examples
   - Two-sided tests vs. one-sided tests

$X_1, X_2, \ldots, X_n$, IID $\mathcal{N}(\mu, \sigma)$

$X_1, X_2, \ldots, X_n$, IID $\mathcal{N}(\mu, \sigma)$ (we sometimes talk about realization ("points") $x_1, \ldots, x_n$ of $X_1, \ldots, X_n$; it will be clear from context which of the two notions we mean)
Sample mean:

$X_1, X_2, \ldots, X_n$, IID $\mathcal{N}(\mu, \sigma)$ (we sometimes talk about realization ("points") $x_1, \ldots, x_n$ of $X_1, \ldots, X_n$; it will be clear from context which of the two notions we mean)

Sample mean:

- Sample mean has realization $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$

$X_1, X_2, \ldots, X_n$, IID $\mathcal{N}(\mu, \sigma)$ (we sometimes talk about realization ("points") $x_1, \ldots, x_n$ of $X_1, \ldots, X_n$; it will be clear from context which of the two notions we mean)

Sample mean:

- Sample mean has realization $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
- $E(\overline{X}) = \mu$ (unbiased)

$X_1, X_2, \ldots, X_n$, IID $\mathcal{N}(\mu, \sigma)$ (we sometimes talk about realization ("points") $x_1, \ldots, x_n$ of $X_1, \ldots, X_n$; it will be clear from context which of the two notions we mean)

Sample mean:

- Sample mean has realization $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
- $E(\overline{X}) = \mu$ (unbiased)
- $V(\overline{X}) = \frac{\sigma^2}{n}$ ($\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$)

$X_1, X_2, \ldots, X_n$, IID $\mathcal{N}(\mu, \sigma)$ (we sometimes talk about realization ("points") $x_1, \ldots, x_n$ of $X_1, \ldots, X_n$; it will be clear from context which of the two notions we mean)

Sample mean:

- Sample mean has realization $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
- $E(\overline{X}) = \mu$ (unbiased)
- $V(\overline{X}) = \frac{\sigma^2}{n}$ ($\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$)

$\Rightarrow$ $\overline{X} \sim \mathcal{N}(\mu, \frac{\sigma}{\sqrt{n}})$

$X_1, X_2, \ldots, X_n$, IID $\mathcal{N}(\mu, \sigma)$ (we sometimes talk about realization ("points") $x_1, \ldots, x_n$ of $X_1, \ldots, X_n$; it will be clear from context which of the two notions we mean)

Sample mean:

- Sample mean has realization $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
- $E(\overline{X}) = \mu$ (unbiased)
- $V(\overline{X}) = \frac{\sigma^2}{n}$ $(\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}})$

$\Rightarrow$ $\overline{X} \sim \mathcal{N}(\mu, \frac{\sigma}{\sqrt{n}})$

Sample variance:

$X_1, X_2, \ldots, X_n$, IID $\mathcal{N}(\mu, \sigma)$ (we sometimes talk about realization ("points") $x_1, \ldots, x_n$ of $X_1, \ldots, X_n$; it will be clear from context which of the two notions we mean)

Sample mean:

- Sample mean has realization $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
- $E(\overline{X}) = \mu$ (unbiased)
- $V(\overline{X}) = \frac{\sigma^2}{n}$ ($\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$)

$\Rightarrow$ $\overline{X} \sim \mathcal{N}(\mu, \frac{\sigma}{\sqrt{n}})$

Sample variance:

- Sample variance $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ (realization $s^2 = \frac{1}{n-1}\sum_{i=1}^{m}(x_i - \bar{x})^2$)

$X_1, X_2, \ldots, X_n$, IID $\mathcal{N}(\mu, \sigma)$ (we sometimes talk about realization ("points") $x_1, \ldots, x_n$ of $X_1, \ldots, X_n$; it will be clear from context which of the two notions we mean)

Sample mean:

- Sample mean has realization $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
- $E(\overline{X}) = \mu$ (unbiased)
- $V(\overline{X}) = \frac{\sigma^2}{n}$ ($\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$)

$\Rightarrow$ $\overline{X} \sim \mathcal{N}(\mu, \frac{\sigma}{\sqrt{n}})$

Sample variance:

- Sample variance $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ (realization $s^2 = \frac{1}{n-1}\sum_{i=1}^{m}(x_i - \bar{x})^2$)
- $E(S^2) = \sigma^2$ (unbiased)

$X_1, X_2, \ldots, X_n$, IID $\mathcal{N}(\mu, \sigma)$ (we sometimes talk about realization ("points") $x_1, \ldots, x_n$ of $X_1, \ldots, X_n$; it will be clear from context which of the two notions we mean)

Sample mean:

- Sample mean has realization $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
- $E(\overline{X}) = \mu$ (unbiased)
- $V(\overline{X}) = \frac{\sigma^2}{n}$ $(\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}})$

$\Rightarrow$ $\overline{X} \sim \mathcal{N}(\mu, \frac{\sigma}{\sqrt{n}})$

Sample variance:

- Sample variance $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ (realization $s^2 = \frac{1}{n-1}\sum_{i=1}^{m}(x_i - \bar{x})^2$)
- $E(S^2) = \sigma^2$ (unbiased)

Last week – estimation of σ^2 or σ

$X_1, X_2, \ldots, X_n$, IID $\mathcal{N}(\mu, \sigma)$ (we sometimes talk about realization ("points") $x_1, \ldots, x_n$ of $X_1, \ldots, X_n$; it will be clear from context which of the two notions we mean)

Sample mean:

- Sample mean has realization $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
- $E(\overline{X}) = \mu$ (unbiased)
- $V(\overline{X}) = \frac{\sigma^2}{n}$ $(\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}})$

$\Rightarrow$ $\overline{X} \sim \mathcal{N}(\mu, \frac{\sigma}{\sqrt{n}})$

Sample variance:

- Sample variance $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ (realization $s^2 = \frac{1}{n-1}\sum_{i=1}^{m}(x_i - \bar{x})^2$)
- $E(S^2) = \sigma^2$ (unbiased)

Last week – estimation of σ^2 or σ

- μ known: $\underline{\chi}^2 \stackrel{\text{def}}{=} \sum_{i=1}^{n} \frac{(X_i - \mu)}{\sigma^2} \sim \chi_n^2$ (chi-squared distribution with n degrees of freedom) $\Rightarrow$ calculate confidence interval (CI) for σ^2

$X_1, X_2, \ldots, X_n$, IID $\mathcal{N}(\mu, \sigma)$ (we sometimes talk about realization ("points") $x_1, \ldots, x_n$ of $X_1, \ldots, X_n$; it will be clear from context which of the two notions we mean)

Sample mean:

- Sample mean has realization $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$
- $E(\overline{X}) = \mu$ (unbiased)
- $V(\overline{X}) = \frac{\sigma^2}{n}$ $(\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}})$

$\Rightarrow$ $\overline{X} \sim \mathcal{N}(\mu, \frac{\sigma}{\sqrt{n}})$

Sample variance:

- Sample variance $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$ (realization $s^2 = \frac{1}{n-1}\sum_{i=1}^{m}(x_i - \bar{x})^2$)
- $E(S^2) = \sigma^2$ (unbiased)

Last week – estimation of σ^2 or σ

- μ known: $\underline{\chi}^2 \stackrel{\text{def}}{=} \sum_{i=1}^{n} \frac{(X_i - \mu)}{\sigma^2} \sim \chi^2_n$ (chi-squared distribution with n degrees of freedom) $\Rightarrow$ calculate confidence interval (CI) for σ^2
- μ unknown: $\underline{\chi}^2 \stackrel{\text{def}}{=} \sum_{i=1}^{n} \frac{(X_i - \overline{X})}{\sigma^2} = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{n-1}$ (chi-squared distribution with n degrees of freedom) $\Rightarrow$ calculate confidence interval (CI) for σ^2 and σ^2 (book: Section 9.12)

And now . . .

1. Revision: Estimation Problems
2. **Revision: Central Limit Theorem**
3. Hypothesis testing
   - Basics
   - Errors
   - Examples
   - Two-sided tests vs. one-sided tests

Central limit theorem (CLT)

- Let $X_1, \ldots, X_n$ be IID with unknown μ and σ.
- Then $E(\overline{X}) = \mu$, $V(\overline{X}) = \frac{\sigma^2}{n}$, and, if n is big enough,

$$Z \stackrel{\text{def}}{=} \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \approx \mathcal{N}(0, 1)$$

Central limit theorem (CLT)

- Let $X_1, \ldots, X_n$ be IID with unknown μ and σ.
- Then $E(\overline{X}) = \mu$, $V(\overline{X}) = \frac{\sigma^2}{n}$, and, if n is big enough,

$$Z \stackrel{\text{def}}{=} \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \approx \mathcal{N}(0, 1)$$

Remarks

Central limit theorem (CLT)

- Let $X_1, \ldots, X_n$ be IID with unknown μ and σ.
- Then $E(\overline{X}) = \mu$, $V(\overline{X}) = \frac{\sigma^2}{n}$, and, if n is big enough,

$$Z \stackrel{\text{def}}{=} \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \approx \mathcal{N}(0, 1)$$

Remarks

- Use T-distribution if you have enough information: i.e. normal distribution and a not too large sample

Central limit theorem (CLT)

- Let $X_1, \ldots, X_n$ be IID with unknown μ and σ.
- Then $E(\overline{X}) = \mu$, $V(\overline{X}) = \frac{\sigma^2}{n}$, and, if n is big enough,

$$Z \stackrel{\text{def}}{=} \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \approx \mathcal{N}(0, 1)$$

Remarks

- Use T-distribution if you have enough information: i.e. normal distribution and a not too large sample
- $\underline{\chi}^2 \stackrel{\text{def}}{=} \frac{(n-1)\, s^2}{\sigma^2} \sim \chi^2_{n-1}$

Central limit theorem (CLT)

- Let $X_1, \ldots, X_n$ be IID with unknown μ and σ.
- Then $E(\overline{X}) = \mu$, $V(\overline{X}) = \frac{\sigma^2}{n}$, and, if n is big enough,

$$Z \stackrel{\text{def}}{=} \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \approx \mathcal{N}(0, 1)$$

Remarks

- Use T-distribution if you have enough information: i.e. normal distribution and a not too large sample
- $\underline{\chi}^2 \stackrel{\text{def}}{=} \frac{(n-1)\, s^2}{\sigma^2} \sim \chi^2_{n-1} \Rightarrow V(\underline{\chi}^2) = 2(n-1)$

Central limit theorem (CLT)

- Let $X_1, \ldots, X_n$ be IID with unknown μ and σ.
- Then $E(\overline{X}) = \mu$, $V(\overline{X}) = \frac{\sigma^2}{n}$, and, if n is big enough,

$$Z \stackrel{\text{def}}{=} \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \approx \mathcal{N}(0, 1)$$

Remarks

- Use T-distribution if you have enough information: i.e. normal distribution and a not too large sample
- $\underline{\chi}^2 \stackrel{\text{def}}{=} \frac{(n-1)\, s^2}{\sigma^2} \sim \chi^2_{n-1} \Rightarrow V(\underline{\chi}^2) = 2(n-1)$

$\Rightarrow$ $V(\frac{s^2}{\sigma^2}) = \frac{1}{(n-1)^2} \cdot V(\underline{\chi}^2) = \frac{2}{n-1} \stackrel{n\to\infty}{\to} 0$, whereas $E(\frac{s^2}{\sigma^2}) = \frac{n-1}{n-1} = 1 \Rightarrow$
That is why we may replace σ by S.

And now . . .

Basics

Null hypothesis

Basics

Null hypothesis

Hypothesis: Some assertion/conjecture concerning the population that we are drawing from. E.g.:

Null hypothesis

Hypothesis: Some assertation/conjecture concerning the population that we are drawing from. E.g.:

(a) $\mu = 20$

Null hypothesis

Hypothesis: Some assertation/conjecture concerning the population that we are drawing from. E.g.:

(a) $\mu = 20$

(b) $\mu \leq 20$

Null hypothesis

Hypothesis: Some assertation/conjecture concerning the population that we are drawing from. E.g.:

(a) $\mu = 20$

(b) $\mu \leq 20$

(c) $\sigma \geq 5$

Null hypothesis

Hypothesis: Some assertion/conjecture concerning the population that we are drawing from. E.g.:

$$\left.\begin{array}{ll} \textbf{(a)} & \mu = 20 \\ \textbf{(b)} & \mu \leq 20 \\ \textbf{(c)} & \sigma \geq 5 \end{array}\right\} H_0 : \text{The null hypothesis}$$

Null hypothesis

Hypothesis: Some assertation/conjecture concerning the population that we are drawing from. E.g.:

$$\left.\begin{array}{ll} \textbf{(a)} & \mu = 20 \\ \textbf{(b)} & \mu \leq 20 \\ \textbf{(c)} & \sigma \geq 5 \end{array}\right\} H_0 : \text{The null hypothesis}$$

Alternative hypothesis

Null hypothesis

Hypothesis: Some assertion/conjecture concerning the population that we are drawing from. E.g.:

$$\left.\begin{array}{ll}\textbf{(a)} & \mu = 20 \\ \textbf{(b)} & \mu \leq 20 \\ \textbf{(c)} & \sigma \geq 5\end{array}\right\} H_0 \text{ : The null hypothesis}$$

Alternative hypothesis

(a) $\mu \neq 20$

Null hypothesis

Hypothesis: Some assertation/conjecture concerning the population that we are drawing from. E.g.:

$$\left.\begin{array}{ll} \textbf{(a)} & \mu = 20 \\ \textbf{(b)} & \mu \leq 20 \\ \textbf{(c)} & \sigma \geq 5 \end{array}\right\} H_0 : \text{The null hypothesis}$$

Alternative hypothesis

(a) $\mu \neq 20$

(b) $\mu > 20$

Null hypothesis

Hypothesis: Some assertation/conjecture concerning the population that we are drawing from. E.g.:

$$\left.\begin{array}{ll}\textbf{(a)} & \mu = 20 \\ \textbf{(b)} & \mu \leq 20 \\ \textbf{(c)} & \sigma \geq 5\end{array}\right\} H_0 \text{ : The null hypothesis}$$

Alternative hypothesis

(a) $\mu \neq 20$

(b) $\mu > 20$

(c) $\sigma < 5$

Null hypothesis

Hypothesis: Some assertation/conjecture concerning the population that we are drawing from. E.g.:

$$\left.\begin{array}{ll}\textbf{(a)} & \mu = 20 \\ \textbf{(b)} & \mu \leq 20 \\ \textbf{(c)} & \sigma \geq 5\end{array}\right\} H_0 \text{ : The null hypothesis}$$

Alternative hypothesis

$$\left.\begin{array}{ll}\textbf{(a)} & \mu \neq 20 \\ \textbf{(b)} & \mu > 20 \\ \textbf{(c)} & \sigma < 5\end{array}\right\} H_1 \text{ : The alternative hypothesis}$$

Null hypothesis

Hypothesis: Some assertation/conjecture concerning the population that we are drawing from. E.g.:

$$\left.\begin{array}{ll}\textbf{(a)} & \mu = 20 \\ \textbf{(b)} & \mu \leq 20 \\ \textbf{(c)} & \sigma \geq 5\end{array}\right\} H_0 : \text{The null hypothesis}$$

Alternative hypothesis

$$\left.\begin{array}{ll}\textbf{(a)} & \mu \neq 20 \\ \textbf{(b)} & \mu > 20 \\ \textbf{(c)} & \sigma < 5\end{array}\right\} H_1 : \text{The alternative hypothesis}$$

Accepting/rejecting H_0

Depending on the realization of the random sample we accept or reject H_0.

Errors

Type 1

Errors

Type 1

Rejection of H_0 when it is actually true; probability α (significance level)

Errors

Type 1

Rejection of H_0 when it is actually true; probability α (significance level)

Type 2

Errors

Type 1

Rejection of H_0 when it is actually true; probability α (significance level)

Type 2

Accepting H_0 when it is actually false; probability β

Examples

Example 1 (a)

Example 1 (a)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$

Examples

Example 1 (a)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$

Example 1 (a)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$

- Note: Under H_0 (if H_0 is true):

Example 1 (a)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$

- Note: Under H_0 (if H_0 is true): $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1$,

Example 1 (a)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$

- Note: Under H_0 (if H_0 is true): $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1$, $\sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1$.

Example 1 (a)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$

- Note: Under H_0 (if H_0 is true): $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1$, $\sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1$.
- We decide to accept H_0 if $\sum_{i=1}^n X_i \in [95, 100]$ - this is acceptance region

Example 1 (a)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$

- Note: Under H_0 (if H_0 is true): $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1$, $\sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1$.
- We decide to accept H_0 if $\sum_{i=1}^{n} X_i \in [95, 100]$ - this is acceptance region (notice that $E(\sum_{i=1}^{n} X_i) = 100$ under H_0)

Example 1 (a)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$

- Note: Under H_0 (if H_0 is true): $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1$, $\sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1$.
- We decide to accept H_0 if $\sum_{i=1}^n X_i \in [95, 100]$ - this is acceptance region (notice that $E(\sum_{i=1}^n X_i) = 100$ under H_0)
- Task: Calculate the significance level α (probability of rejecting H_0 when it is true)

Example 1 (a)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$

- Note: Under H_0 (if H_0 is true): $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1$, $\sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1$.
- We decide to accept H_0 if $\sum_{i=1}^{n} X_i \in [95, 100]$ - this is acceptance region (notice that $E(\sum_{i=1}^{n} X_i) = 100$ under H_0)
- Task: Calculate the significance level α (probability of rejecting H_0 when it is true)
- Solution: $Z \overset{\text{def}}{=} \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \approx \mathcal{N}(0,1)$ (CLT)

Example 1 (a)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution ($\mathrm{Exp}(\lambda)$, $\mathrm{Exp}(\beta)$). We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$

- Note: Under H_0 (if H_0 is true): $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1$, $\sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1$.
- We decide to accept H_0 if $\sum_{i=1}^n X_i \in [95, 100]$ - this is acceptance region (notice that $E(\sum_{i=1}^n X_i) = 100$ under H_0)
- Task: Calculate the significance level α (probability of rejecting H_0 when it is true)
- Solution: $Z \stackrel{\text{def}}{=} \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \approx \mathcal{N}(0,1)$ (CLT)
- Furthermore: $\sum_{i=1}^n X_i \in [95, 105] \Leftrightarrow \bar{X} = \frac{1}{n}\sum_{i=1}^n X_i \in [0.95, 1.05]$

Example 1 (a)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$

- Note: Under H_0 (if H_0 is true): $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1$, $\sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1$.
- We decide to accept H_0 if $\sum_{i=1}^{n} X_i \in [95, 100]$ - this is acceptance region (notice that $E(\sum_{i=1}^{n} X_i) = 100$ under H_0)
- Task: Calculate the significance level α (probability of rejecting H_0 when it is true)
- Solution: $Z \stackrel{\text{def}}{=} \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \approx \mathcal{N}(0,1)$ (CLT)
- Furthermore: $\sum_{i=1}^{n} X_i \in [95, 105] \Leftrightarrow \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \in [0.95, 1.05]$
- Under $H_0 : \mu = 1, \sigma = 1 \Rightarrow Z \stackrel{\text{def}}{=} \frac{\bar{X}-1}{1/\sqrt{100}} \approx \mathcal{N}(0,1)$ and:

Example 1 (a)

Let X_1, X_2, ..., X_{100} IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$

- Note: Under H_0 (if H_0 is true): $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1$, $\sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1$.
- We decide to accept H_0 if $\sum_{i=1}^n X_i \in [95, 100]$ - this is acceptance region (notice that $E(\sum_{i=1}^n X_i) = 100$ under H_0)
- Task: Calculate the significance level α (probability of rejecting H_0 when it is true)
- Solution: $Z \stackrel{\text{def}}{=} \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \approx \mathcal{N}(0,1)$ (CLT)
- Furthermore: $\sum_{i=1}^n X_i \in [95, 105] \Leftrightarrow \bar{X} = \frac{1}{n}\sum_{i=1}^n X_i \in [0.95, 1.05]$
- Under $H_0 : \mu = 1, \sigma = 1 \Rightarrow Z \stackrel{\text{def}}{=} \frac{\bar{X}-1}{1/\sqrt{100}} \approx \mathcal{N}(0,1)$ and:

$$\alpha = P(\sum_{i=1}^n X_i < 95) + P(\sum_{i=1}^n X_i > 105)$$

Example 1 (a)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$

- Note: Under H_0 (if H_0 is true): $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1$, $\sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1$.
- We decide to accept H_0 if $\sum_{i=1}^n X_i \in [95, 100]$ - this is acceptance region (notice that $E(\sum_{i=1}^n X_i) = 100$ under H_0)
- Task: Calculate the significance level α (probability of rejecting H_0 when it is true)
- Solution: $Z \stackrel{\text{def}}{=} \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \approx \mathcal{N}(0,1)$ (CLT)
- Furthermore: $\sum_{i=1}^n X_i \in [95, 105] \Leftrightarrow \bar{X} = \frac{1}{n}\sum_{i=1}^n X_i \in [0.95, 1.05]$
- Under $H_0 : \mu = 1, \sigma = 1 \Rightarrow Z \stackrel{\text{def}}{=} \frac{\bar{X}-1}{1/\sqrt{100}} \approx \mathcal{N}(0,1)$ and:

$$\alpha = P(\sum_{i=1}^n X_i < 95) + P(\sum_{i=1}^n X_i > 105) = P(\overline{X} < 0.95) + P(\overline{X} > 1.05)$$

Example 1 (a)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$

- Note: Under H_0 (if H_0 is true): $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1$, $\sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1$.
- We decide to accept H_0 if $\sum_{i=1}^n X_i \in [95, 100]$ - this is acceptance region (notice that $E(\sum_{i=1}^n X_i) = 100$ under H_0)
- Task: Calculate the significance level α (probability of rejecting H_0 when it is true)
- Solution: $Z \stackrel{\text{def}}{=} \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \approx \mathcal{N}(0, 1)$ (CLT)
- Furthermore: $\sum_{i=1}^n X_i \in [95, 105] \Leftrightarrow \bar{X} = \frac{1}{n}\sum_{i=1}^n X_i \in [0.95, 1.05]$
- Under $H_0 : \mu = 1, \sigma = 1 \Rightarrow Z \stackrel{\text{def}}{=} \frac{\bar{X}-1}{1/\sqrt{100}} \approx \mathcal{N}(0, 1)$ and:

$$\alpha = P(\sum_{i=1}^n X_i < 95) + P(\sum_{i=1}^n X_i > 105) = P(\overline{X} < 0.95) + P(\overline{X} > 1.05)$$

$$= P(Z = \frac{\overline{X} - 1}{1/\sqrt{100}} < \frac{0.95 - 1}{1/\sqrt{100}}) + P(\overline{Z} > \frac{1.05 - 1}{1/\sqrt{100}})$$

Example 1 (a)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$

- Note: Under H_0 (if H_0 is true): $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1$, $\sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1$.
- We decide to accept H_0 if $\sum_{i=1}^n X_i \in [95, 100]$ - this is acceptance region (notice that $E(\sum_{i=1}^n X_i) = 100$ under H_0)
- Task: Calculate the significance level α (probability of rejecting H_0 when it is true)
- Solution: $Z \stackrel{\text{def}}{=} \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \approx \mathcal{N}(0,1)$ (CLT)
- Furthermore: $\sum_{i=1}^n X_i \in [95, 105] \Leftrightarrow \bar{X} = \frac{1}{n}\sum_{i=1}^n X_i \in [0.95, 1.05]$
- Under $H_0 : \mu = 1, \sigma = 1 \Rightarrow Z \stackrel{\text{def}}{=} \frac{\bar{X}-1}{1/\sqrt{100}} \approx \mathcal{N}(0,1)$ and:

$$\alpha = P(\sum_{i=1}^n X_i < 95) + P(\sum_{i=1}^n X_i > 105) = P(\overline{X} < 0.95) + P(\overline{X} > 1.05)$$

$$= P(Z = \frac{\overline{X}-1}{1/\sqrt{100}} < \frac{0.95-1}{1/\sqrt{100}}) + P(\overline{Z} > \frac{1.05-1}{1/\sqrt{100}}) = 0.6170$$

Examples

Example 1 (b)

Examples

Example 1 (b)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda)$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$ With significance level of 0.1, for what outcomes of $\bar{X}$ do we accept H_0?

Example 1 (b)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda)$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$ With significance level of 0.1, for what outcomes of $\bar{X}$ do we accept H_0?

- Under H_0 (meaning: if H_0 is true):

Example 1 (b)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda)$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$ With significance level of 0.1, for what outcomes of $\bar{X}$ do we accept H_0?

- Under H_0 (meaning: if H_0 is true): $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1$

Example 1 (b)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda)$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$ With significance level of 0.1, for what outcomes of $\bar{X}$ do we accept H_0?

- Under H_0 (meaning: if H_0 is true):
  $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1 \quad \sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1,$

Example 1 (b)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda)$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$ With significance level of 0.1, for what outcomes of $\bar{X}$ do we accept H_0?

- Under H_0 (meaning: if H_0 is true):
  $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1 \quad \sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1,$
  $Z = \frac{\overline{X}-1}{1/\sqrt{100}} \sim \mathcal{N}(0, 1)$

Example 1 (b)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda)$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$ With significance level of 0.1, for what outcomes of $\bar{X}$ do we accept H_0?

- Under H_0 (meaning: if H_0 is true):
  $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1 \quad \sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1$,
  $Z = \frac{\overline{X} - 1}{1/\sqrt{100}} \sim \mathcal{N}(0, 1)$
- Find z such that $P(-z \leq Z \leq z) = 1 - 0.10 = 0.90$

Example 1 (b)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda)$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$ With significance level of 0.1, for what outcomes of $\bar{X}$ do we accept H_0?

- Under H_0 (meaning: if H_0 is true):
  $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1 \quad \sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1,$
  $Z = \frac{\overline{X}-1}{1/\sqrt{100}} \sim \mathcal{N}(0,1)$
- Find z such that $P(-z \leq Z \leq z) = 1 - 0.10 = 0.90$
  Table A3 $\Rightarrow P\left(-1.645 \leq \frac{\overline{X}-1}{1/\sqrt{100}} \leq 1.645\right) = 0.90$

Example 1 (b)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda)$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$ With significance level of 0.1, for what outcomes of $\bar{X}$ do we accept H_0?

- Under H_0 (meaning: if H_0 is true):
  $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1 \quad \sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1,$
  $Z = \frac{\bar{X}-1}{1/\sqrt{100}} \sim \mathcal{N}(0,1)$
- Find z such that $P(-z \leq Z \leq z) = 1 - 0.10 = 0.90$
  Table A3 $\Rightarrow P\left(-1.645 \leq \frac{\bar{X}-1}{1/\sqrt{100}} \leq 1.645\right) = 0.90$

$\Leftrightarrow \quad P\left(-1.645 \leq \frac{\bar{X}-1}{1/\sqrt{100}} \leq 1.645\right) = 0.90$

Example 1 (b)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda)$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$ With significance level of 0.1, for what outcomes of $\bar{X}$ do we accept H_0?

- Under H_0 (meaning: if H_0 is true):
  $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1 \quad \sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1,$
  $Z = \frac{\bar{X}-1}{1/\sqrt{100}} \sim \mathcal{N}(0,1)$
- Find z such that $P(-z \leq Z \leq z) = 1 - 0.10 = 0.90$
  Table A3 $\Rightarrow P\left(-1.645 \leq \frac{\bar{X}-1}{1/\sqrt{100}} \leq 1.645\right) = 0.90$

$$\Leftrightarrow \quad P\left(-1.645 \leq \frac{\bar{X}-1}{1/\sqrt{100}} \leq 1.645\right) = 0.90$$

$$\Leftrightarrow \quad P\left(\bar{X} \in [0.83555, 1.1645]\right) = 0.90$$

Example 1 (b)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\text{Exp}(\lambda)$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$ With significance level of 0.1, for what outcomes of $\bar{X}$ do we accept H_0?

- Under H_0 (meaning: if H_0 is true):
  $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1 \quad \sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1,$
  $Z = \frac{\bar{X}-1}{1/\sqrt{100}} \sim \mathcal{N}(0,1)$
- Find z such that $P(-z \leq Z \leq z) = 1 - 0.10 = 0.90$
  Table A3 $\Rightarrow P\left(-1.645 \leq \frac{\bar{X}-1}{1/\sqrt{100}} \leq 1.645\right) = 0.90$

$\Leftrightarrow \quad P\left(-1.645 \leq \frac{\bar{X}-1}{1/\sqrt{100}} \leq 1.645\right) = 0.90$

$\Leftrightarrow \quad P\left(\bar{X} \in [0.83555, 1.1645]\right) = 0.90$

Acceptance region for $H_0 : \lambda = 1$ is $[0.83555, 1.1645]$

Example 1 (b)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda)$. We want to test $H_0 : \lambda = 1$ vs. $H_1 : \lambda \neq 1$ With significance level of 0.1, for what outcomes of $\bar{X}$ do we accept H_0?

- Under H_0 (meaning: if H_0 is true):
  $\mu_X = E(X_i) = \frac{1}{\lambda} = \beta = 1 \quad \sigma_X^2 = V(X_i) = \frac{1}{\lambda^2} = 1,$
  $Z = \frac{\bar{X}-1}{1/\sqrt{100}} \sim \mathcal{N}(0,1)$
- Find z such that $P(-z \leq Z \leq z) = 1 - 0.10 = 0.90$
  Table A3 $\Rightarrow P\left(-1.645 \leq \frac{\bar{X}-1}{1/\sqrt{100}} \leq 1.645\right) = 0.90$

$$\Leftrightarrow \quad P\left(-1.645 \leq \frac{\bar{X}-1}{1/\sqrt{100}} \leq 1.645\right) = 0.90$$

$$\Leftrightarrow \quad P\left(\bar{X} \in [0.83555, 1.1645]\right) = 0.90$$

Acceptance region for $H_0 : \lambda = 1$ is $[0.83555, 1.1645]$
Critical region for H_0 : All other outcomes for $\bar{X}$

Examples

Example 1 (c)

Examples

Example 1 (c)

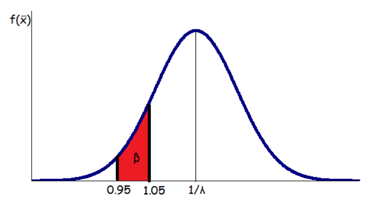
Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$.
Calculate β if in fact $\lambda = 0.85$

Example 1 (c)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$.
Calculate β if in fact $\lambda = 0.85$
In order to be able to calculate β we need a concrete value for the alternative hypothesis. Just $\lambda \neq 1$ is not enough

Example 1 (c)

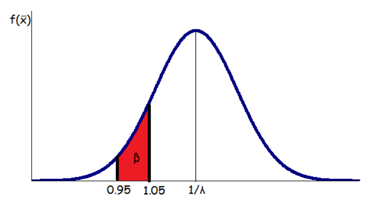
Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$.
Calculate β if in fact $\lambda = 0.85$
In order to be able to calculate β we need a concrete value for the alternative hypothesis. Just $\lambda \neq 1$ is not enough



We accept H_0 if $\bar{x} \in [0.95, 1.05]$

Example 1 (c)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$.
Calculate β if in fact $\lambda = 0.85$
In order to be able to calculate β we need a concrete value for the alternative hypothesis. Just $\lambda \neq 1$ is not enough

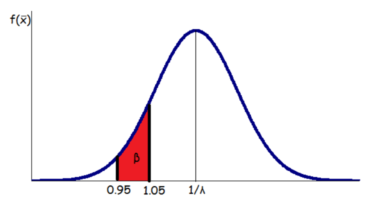


We accept H_0 if $\bar{x} \in [0.95, 1.05]$

Under $H_1 : \lambda = 0.85$ we have (because the distribution is exponential)

Example 1 (c)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$.
Calculate β if in fact $\lambda = 0.85$
In order to be able to calculate β we need a concrete value for the alternative hypothesis. Just $\lambda \neq 1$ is not enough



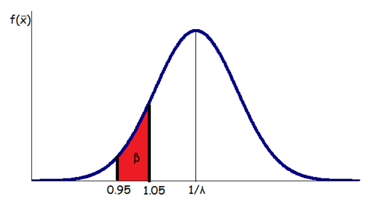
We accept H_0 if $\bar{x} \in [0.95, 1.05]$

Under $H_1 : \lambda = 0.85$ we have (because the distribution is exponential)

- $\mu = E(X_i) = \frac{1}{\lambda} = \frac{1}{0.85}, \sigma^2 = V(X_i) = \frac{1}{\lambda^2} = \frac{1}{0.85^2}$

Example 1 (c)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$.
Calculate β if in fact $\lambda = 0.85$
In order to be able to calculate β we need a concrete value for the alternative hypothesis. Just $\lambda \neq 1$ is not enough



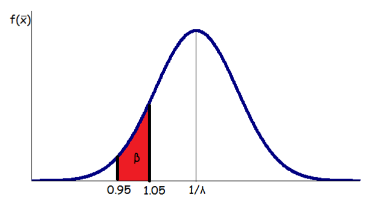
We accept H_0 if $\bar{x} \in [0.95, 1.05]$

Under $H_1 : \lambda = 0.85$ we have (because the distribution is exponential)

- $\mu = E(X_i) = \frac{1}{\lambda} = \frac{1}{0.85}$, $\sigma^2 = V(X_i) = \frac{1}{\lambda^2} = \frac{1}{0.85^2}$
  $Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{\overline{X} - 1/0.85}{\frac{1/0.85}{\sqrt{100}}} \overset{\text{approx}}{\sim} \mathcal{N}(0,1)$ (CLT)

Example 1 (c)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$.
Calculate β if in fact $\lambda = 0.85$
In order to be able to calculate β we need a concrete value for the alternative hypothesis. Just $\lambda \neq 1$ is not enough



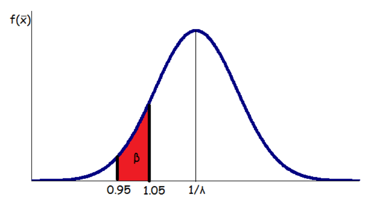
We accept H_0 if $\bar{x} \in [0.95, 1.05]$

Under $H_1 : \lambda = 0.85$ we have (because the distribution is exponential)

- $\mu = E(X_i) = \frac{1}{\lambda} = \frac{1}{0.85}$, $\sigma^2 = V(X_i) = \frac{1}{\lambda^2} = \frac{1}{0.85^2}$
  $Z = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} = \frac{\overline{X}-1/0.85}{\frac{1/0.85}{\sqrt{100}}} \overset{\text{approx}}{\sim} \mathcal{N}(0,1)$ (CLT)
- $\beta = P\left(95 \leq \sum_{i=1}^{n} X_i \leq 105\right)$

Example 1 (c)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$.
Calculate β if in fact $\lambda = 0.85$
In order to be able to calculate β we need a concrete value for the alternative hypothesis. Just $\lambda \neq 1$ is not enough



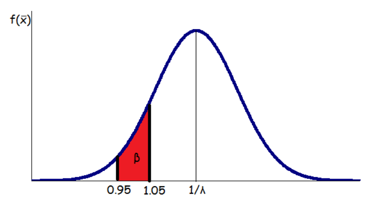
We accept H_0 if $\bar{x} \in [0.95, 1.05]$

Under $H_1 : \lambda = 0.85$ we have (because the distribution is exponential)

- $\mu = E(X_i) = \frac{1}{\lambda} = \frac{1}{0.85}$, $\sigma^2 = V(X_i) = \frac{1}{\lambda^2} = \frac{1}{0.85^2}$
  $Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{\overline{X} - 1/0.85}{\frac{1/0.85}{\sqrt{100}}} \overset{\text{approx}}{\sim} \mathcal{N}(0, 1)$ (CLT)
- $\beta = P\left(95 \leq \sum_{i=1}^{n} X_i \leq 105\right) = P(0.95 \leq \bar{X} \leq 1.05)$

Example 1 (c)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$.
Calculate β if in fact $\lambda = 0.85$
In order to be able to calculate β we need a concrete value for the alternative hypothesis. Just $\lambda \neq 1$ is not enough



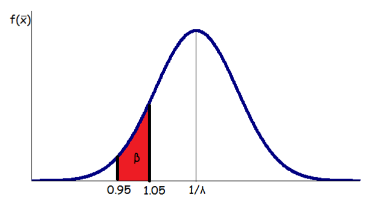
We accept H_0 if $\bar{x} \in [0.95, 1.05]$

Under $H_1 : \lambda = 0.85$ we have (because the distribution is exponential)

- $\mu = E(X_i) = \frac{1}{\lambda} = \frac{1}{0.85}$, $\sigma^2 = V(X_i) = \frac{1}{\lambda^2} = \frac{1}{0.85^2}$
  $Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} = \frac{\overline{X} - 1/0.85}{\frac{1/0.85}{\sqrt{100}}} \overset{\text{approx}}{\sim} \mathcal{N}(0,1)$ (CLT)
- $\beta = P\left(95 \leq \sum_{i=1}^{n} X_i \leq 105\right) = P(0.95 \leq \bar{X} \leq 1.05)$
  $= P\left(\frac{0.95 - 1/0.85}{\frac{1}{0.85}\sqrt{100}} \leq Z \leq \frac{1.05 - 1/0.85}{\frac{1}{0.85}\sqrt{100}}\right)$

Example 1 (c)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$.
Calculate β if in fact $\lambda = 0.85$
In order to be able to calculate β we need a concrete value for the alternative hypothesis. Just $\lambda \neq 1$ is not enough



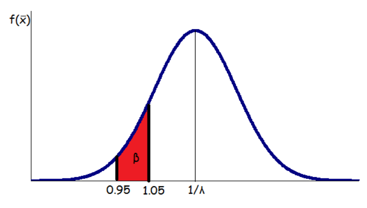
We accept H_0 if $\bar{x} \in [0.95, 1.05]$

Under $H_1 : \lambda = 0.85$ we have (because the distribution is exponential)

- $\mu = E(X_i) = \frac{1}{\lambda} = \frac{1}{0.85}, \sigma^2 = V(X_i) = \frac{1}{\lambda^2} = \frac{1}{0.85^2}$
  $Z = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} = \frac{\overline{X}-1/0.85}{\frac{1/0.85}{\sqrt{100}}} \overset{\text{approx}}{\sim} \mathcal{N}(0,1)$ (CLT)
- $\beta = P\left(95 \leq \sum_{i=1}^{n} X_i \leq 105\right) = P(0.95 \leq \bar{X} \leq 1.05)$
  $= P\left(\frac{0.95-1/0.85}{\frac{1}{0.85}\sqrt{100}} \leq Z \leq \frac{1.05-1/0.85}{\frac{1}{0.85}\sqrt{100}}\right) = P(-1.925 \leq Z \leq -1.075)$

Example 1 (c)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\text{Exp}(\lambda), \text{Exp}(\beta))$.
Calculate β if in fact $\lambda = 0.85$
In order to be able to calculate β we need a concrete value for the alternative hypothesis. Just $\lambda \neq 1$ is not enough



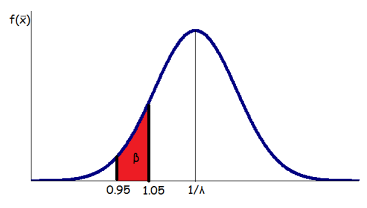
We accept H_0 if $\bar{x} \in [0.95, 1.05]$

Under $H_1 : \lambda = 0.85$ we have (because the distribution is exponential)

- $\mu = E(X_i) = \frac{1}{\lambda} = \frac{1}{0.85}$, $\sigma^2 = V(X_i) = \frac{1}{\lambda^2} = \frac{1}{0.85^2}$
  $Z = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} = \frac{\overline{X}-1/0.85}{\frac{1/0.85}{\sqrt{100}}} \overset{\text{approx}}{\sim} \mathcal{N}(0,1)$ (CLT)
- $\beta = P\left(95 \leq \sum_{i=1}^{n} X_i \leq 105\right) = P(0.95 \leq \bar{X} \leq 1.05)$
  $= P\left(\frac{0.95-1/0.85}{\frac{1}{0.85}\sqrt{100}} \leq Z \leq \frac{1.05-1/0.85}{\frac{1}{0.85}\sqrt{100}}\right) = P(-1.925 \leq Z \leq -1.075)$
  $= 0.1412 - 0.0271$

Example 1 (c)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$.
Calculate β if in fact $\lambda = 0.85$
In order to be able to calculate β we need a concrete value for the alternative hypothesis. Just $\lambda \neq 1$ is not enough



We accept H_0 if $\bar{x} \in [0.95, 1.05]$

Under $H_1 : \lambda = 0.85$ we have (because the distribution is exponential)

- $\mu = E(X_i) = \frac{1}{\lambda} = \frac{1}{0.85}$, $\sigma^2 = V(X_i) = \frac{1}{\lambda^2} = \frac{1}{0.85^2}$
  $Z = \frac{\overline{X}-\mu}{\sigma/\sqrt{n}} = \frac{\overline{X}-1/0.85}{\frac{1/0.85}{\sqrt{100}}} \overset{\text{approx}}{\sim} \mathcal{N}(0,1)$ (CLT)
- $\beta = P\left(95 \leq \sum_{i=1}^{n} X_i \leq 105\right) = P(0.95 \leq \bar{X} \leq 1.05)$
  $= P\left(\frac{0.95-1/0.85}{\frac{1}{0.85}\sqrt{100}} \leq Z \leq \frac{1.05-1/0.85}{\frac{1}{0.85}\sqrt{100}}\right) = P(-1.925 \leq Z \leq -1.075)$
  $= 0.1412 - 0.0271 = 0.1141$

Examples

Example 1 (d)

Example 1 (d)

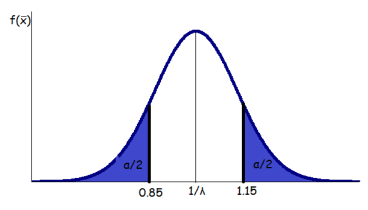
Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We found $\overline{X} = 1.15$. For what levels of significance do we accept $H_0 : \lambda = 1$?

Example 1 (d)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We found $\overline{X} = 1.15$. For what levels of significance do we accept $H_0 : \lambda = 1$? We calculate a significance level such that $\overline{X} = 1.15$ would just be accepted or just be rejected. This significance level is called the *P*-value

Example 1 (d)

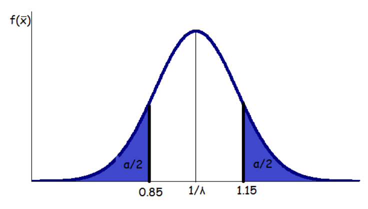
Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We found $\overline{X} = 1.15$. For what levels of significance do we accept $H_0 : \lambda = 1$?
We calculate a significance level such that $\overline{X} = 1.15$ would just be accepted or just be rejected. This significance level is called the *P*-value
$H_0 = 1$ vs. $H_1 : \lambda \neq 1$



$f(\bar{x})$ under H_0

Example 1 (d)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We found $\overline{X} = 1.15$. For what levels of significance do we accept $H_0 : \lambda = 1$?
We calculate a significance level such that $\overline{X} = 1.15$ would just be accepted or just be rejected. This significance level is called the *P*-value
$H_0 = 1$ vs. $H_1 : \lambda \neq 1$

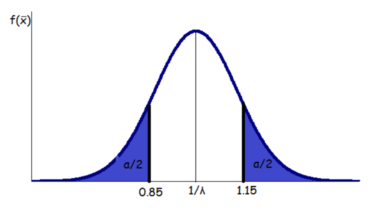


$f(\bar{x})$ under H_0

$$P\text{-value} \stackrel{\text{def}}{=} P(\overline{X} \leq 0.85 \quad \text{or} \quad \overline{X} \geq 1.15)$$

Example 1 (d)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We found $\overline{X} = 1.15$. For what levels of significance do we accept $H_0 : \lambda = 1$?
We calculate a significance level such that $\overline{X} = 1.15$ would just be accepted or just be rejected. This significance level is called the *P*-value
$H_0 = 1$ vs. $H_1 : \lambda \neq 1$

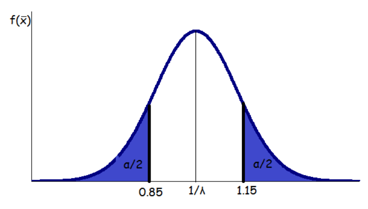


$f(\bar{x})$ under H_0

$P\text{-value} \stackrel{\text{def}}{=} P(\overline{X} \leq 0.85 \quad \text{or} \quad \overline{X} \geq 1.15) = 2\,P(\overline{X} \leq 0.85)$

Example 1 (d)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We found $\overline{X} = 1.15$. For what levels of significance do we accept $H_0 : \lambda = 1$?
We calculate a significance level such that $\overline{X} = 1.15$ would just be accepted or just be rejected. This significance level is called the *P*-value
$H_0 = 1$ vs. $H_1 : \lambda \neq 1$



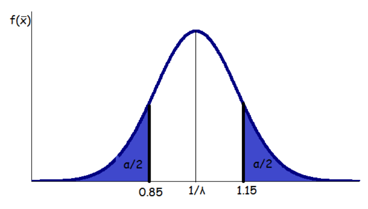
$f(\bar{x})$ under H_0

$$P\text{-value} \stackrel{\text{def}}{=} P(\overline{X} \leq 0.85 \quad \text{or} \quad \overline{X} \geq 1.15) = 2\,P(\overline{X} \leq 0.85) = 2\,P(Z \leq \tfrac{0.85-1}{1/\sqrt{100}})$$

Examples

Example 1 (d)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We found $\overline{X} = 1.15$. For what levels of significance do we accept $H_0 : \lambda = 1$?
We calculate a significance level such that $\overline{X} = 1.15$ would just be accepted or just be rejected. This significance level is called the *P*-value
$H_0 = 1$ vs. $H_1 : \lambda \neq 1$

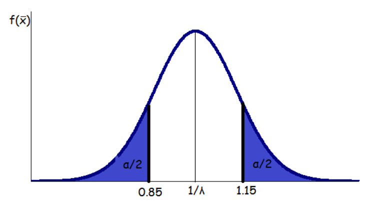


$f(\bar{x})$ under H_0

$$P\text{-value} \stackrel{\text{def}}{=} P(\overline{X} \le 0.85 \quad \text{or} \quad \overline{X} \ge 1.15) = 2\,P(\overline{X} \le 0.85) = 2\,P(Z \le \frac{0.85-1}{1/\sqrt{100}})$$
$$2 \cdot P(Z \le -1.5) = 2 \cdot 0.0668 = 0.1336$$

Example 1 (d)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\mathrm{Exp}(\lambda), \mathrm{Exp}(\beta))$. We found $\overline{X} = 1.15$. For what levels of significance do we accept $H_0 : \lambda = 1$?
We calculate a significance level such that $\overline{X} = 1.15$ would just be accepted or just be rejected. This significance level is called the *P*-value
$H_0 = 1$ vs. $H_1 : \lambda \neq 1$

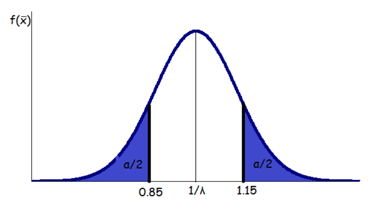


$f(\bar{x})$ under H_0

$P\text{-value} \stackrel{\text{def}}{=} P(\overline{X} \leq 0.85 \quad \text{or} \quad \overline{X} \geq 1.15) = 2\,P(\overline{X} \leq 0.85) = 2\,P(Z \leq \frac{0.85-1}{1/\sqrt{100}})$
$2 \cdot P(Z \leq -1.5) = 2 \cdot 0.0668 = 0.1336$
So, there is a probability of 0.1336 that $\bar{X}$ differs at least 0.15 from μ.

Example 1 (d)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\text{Exp}(\lambda), \text{Exp}(\beta))$. We found $\overline{X} = 1.15$. For what levels of significance do we accept $H_0 : \lambda = 1$?
We calculate a significance level such that $\overline{X} = 1.15$ would just be accepted or just be rejected. This significance level is called the *P*-value
$H_0 = 1$ vs. $H_1 : \lambda \neq 1$

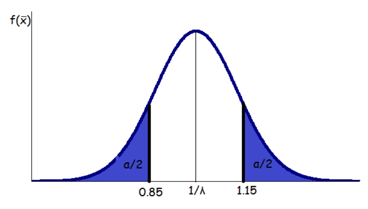


$f(\bar{x})$ under H_0

P-value $\stackrel{\text{def}}{=} P(\overline{X} \leq 0.85 \quad \text{or} \quad \overline{X} \geq 1.15) = 2\,P(\overline{X} \leq 0.85) = 2\,P(Z \leq \frac{0.85-1}{1/\sqrt{100}})$
$2 \cdot P(Z \leq -1.5) = 2 \cdot 0.0668 = 0.1336$
So, there is a probability of 0.1336 that $\bar{X}$ differs at least 0.15 from μ.
What is the conclusion from this observation regarding H_0?

Example 1 (d)

Let $X_1, X_2, \ldots, X_{100}$ IID from exponential distribution $(\text{Exp}(\lambda), \text{Exp}(\beta))$. We found $\overline{X} = 1.15$. For what levels of significance do we accept $H_0 : \lambda = 1$?
We calculate a significance level such that $\overline{X} = 1.15$ would just be accepted or just be rejected. This significance level is called the *P*-value
$H_0 = 1$ vs. $H_1 : \lambda \neq 1$



$f(\bar{x})$ under H_0

P-value $\stackrel{\text{def}}{=} P(\overline{X} \leq 0.85 \quad \text{or} \quad \overline{X} \geq 1.15) = 2\,P(\overline{X} \leq 0.85) = 2\,P(Z \leq \frac{0.85-1}{1/\sqrt{100}})$
$2 \cdot P(Z \leq -1.5) = 2 \cdot 0.0668 = 0.1336$
So, there is a probability of 0.1336 that $\bar{X}$ differs at least 0.15 from μ.
What is the conclusion from this observation regarding H_0?

P-value: The highest α such that H_0 is accepted and the lowest α such that H_0 is rejected.

Two-sided tests vs. one-sided tests

One-sided vs. two-sided tests

One-sided vs. two-sided tests

So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)

One-sided vs. two-sided tests

So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)
Now: $H_0 : \mu = x$ vs. $H_1 : \mu > x$ or $H_1 : \mu < x$ (so-called one-sided tests)

One-sided vs. two-sided tests

So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)
Now: $H_0 : \mu = x$ vs. $H_1 : \mu > x$ or $H_1 : \mu < x$ (so-called one-sided tests)
Same as $H_0 : \mu \geq x$ vs. $H_1 : \mu < x$, or $H_0 : \mu \leq x$ vs. $H_1 : \mu > x$

One-sided vs. two-sided tests

So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)
Now: $H_0 : \mu = x$ vs. $H_1 : \mu > x$ or $H_1 : \mu < x$ (so-called one-sided tests)
Same as $H_0 : \mu \geq x$ vs. $H_1 : \mu < x$, or $H_0 : \mu \leq x$ vs. $H_1 : \mu > x$
We use one-sided tests for calculation purposes: Given that $\mu = x$ we use CLT and/or probability distributions

One-sided vs. two-sided tests

So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)
Now: $H_0 : \mu = x$ vs. $H_1 : \mu > x$ or $H_1 : \mu < x$ (so-called one-sided tests)
Same as $H_0 : \mu \geq x$ vs. $H_1 : \mu < x$, or $H_0 : \mu \leq x$ vs. $H_1 : \mu > x$
We use one-sided tests for calculation purposes: Given that $\mu = x$ we use CLT and/or probability distributions

- $H_o : \mu \geq x$

One-sided vs. two-sided tests

So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)
Now: $H_0 : \mu = x$ vs. $H_1 : \mu > x$ or $H_1 : \mu < x$ (so-called one-sided tests)
Same as $H_0 : \mu \geq x$ vs. $H_1 : \mu < x$, or $H_0 : \mu \leq x$ vs. $H_1 : \mu > x$
We use one-sided tests for calculation purposes: Given that $\mu = x$ we use CLT and/or probability distributions

- $H_o : \mu \geq x$
- Acceptance region: $\bar{X} \geq y$

One-sided vs. two-sided tests

So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)
Now: $H_0 : \mu = x$ vs. $H_1 : \mu > x$ or $H_1 : \mu < x$ (so-called one-sided tests)
Same as $H_0 : \mu \geq x$ vs. $H_1 : \mu < x$, or $H_0 : \mu \leq x$ vs. $H_1 : \mu > x$
We use one-sided tests for calculation purposes: Given that $\mu = x$ we use CLT and/or probability distributions

- $H_o : \mu \geq x$
- Acceptance region: $\bar{X} \geq y$
- Significance level: $\alpha = P(\overline{X} \leq y)$

One-sided vs. two-sided tests

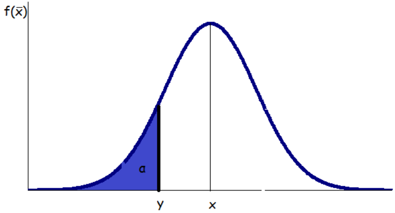
So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)
Now: $H_0 : \mu = x$ vs. $H_1 : \mu > x$ or $H_1 : \mu < x$ (so-called one-sided tests)
Same as $H_0 : \mu \geq x$ vs. $H_1 : \mu < x$, or $H_0 : \mu \leq x$ vs. $H_1 : \mu > x$
We use one-sided tests for calculation purposes: Given that $\mu = x$ we use CLT and/or probability distributions

- $H_o : \mu \geq x$
- Acceptance region: $\bar{X} \geq y$
- Significance level: $\alpha = P(\overline{X} \leq y)$
- P-value also one-sided

One-sided vs. two-sided tests

So far: $H_0 : \mu = x$ vs. $H_1 : \mu \neq x$ (so-called two-sided tests)
Now: $H_0 : \mu = x$ vs. $H_1 : \mu > x$ or $H_1 : \mu < x$ (so-called one-sided tests)
Same as $H_0 : \mu \geq x$ vs. $H_1 : \mu < x$, or $H_0 : \mu \leq x$ vs. $H_1 : \mu > x$
We use one-sided tests for calculation purposes: Given that $\mu = x$ we use CLT and/or probability distributions

- $H_o : \mu \geq x$
- Acceptance region: $\bar{X} \geq y$
- Significance level: $\alpha = P(\overline{X} \leq y)$
- P-value also one-sided



Two-sided tests vs. one-sided tests

Example 2 (a)

Two-sided tests vs. one-sided tests

Example 2 (a)

$X_1, \ldots, X_5 \sim \mathrm{Exp}(\lambda)$, IID, λ unknown

Two-sided tests vs. one-sided tests

Example 2 (a)

$X_1, \ldots, X_5 \sim \text{Exp}(\lambda)$, IID, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$

Two-sided tests vs. one-sided tests

Example 2 (a)

$X_1, \ldots, X_5 \sim \mathrm{Exp}(\lambda)$, IID, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
So under $H_0 : \mu = E(X_i) = \frac{1}{2}$

Example 2 (a)

$X_1, \ldots, X_5 \sim \mathrm{Exp}(\lambda)$, IID, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
So under $H_0 : \mu = E(X_i) = \frac{1}{2}$
Assume that we decide to accept H_0 if $\bar{X} > 0.45$; calculate α

Example 2 (a)

$X_1, \ldots, X_5 \sim \mathrm{Exp}(\lambda)$, IID, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
So under $H_0 : \mu = E(X_i) = \frac{1}{2}$
Assume that we decide to accept H_0 if $\bar{X} > 0.45$; calculate α

- $\bar{X} > 0.45 \Leftrightarrow \sum_{i=1}^{5} X_i > 5 \cdot 0.45 = 2.25$

Two-sided tests vs. one-sided tests

Example 2 (a)

$X_1, \ldots, X_5 \sim \mathrm{Exp}(\lambda)$, IID, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
So under $H_0 : \mu = E(X_i) = \frac{1}{2}$
Assume that we decide to accept H_0 if $\bar{X} > 0.45$; calculate α

- $\bar{X} > 0.45 \Leftrightarrow \sum_{i=1}^{5} X_i > 5 \cdot 0.45 = 2.25$
  Let $Y = \sum_{i=1}^{5} X_i$ (sum of independent exponentially distributed RV's)

Example 2 (a)

$X_1, \ldots, X_5 \sim \mathrm{Exp}(\lambda)$, IID, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
So under $H_0 : \mu = E(X_i) = \frac{1}{2}$
Assume that we decide to accept H_0 if $\bar{X} > 0.45$; calculate α

- $\bar{X} > 0.45 \Leftrightarrow \sum_{i=1}^{5} X_i > 5 \cdot 0.45 = 2.25$
  Let $Y = \sum_{i=1}^{5} X_i$ (sum of independent exponentially distributed RV's)

$\Rightarrow$ $Y \sim \mathrm{Erl}(\lambda = 2, n = 5)$ (under H_0),

Example 2 (a)

$X_1, \ldots, X_5 \sim \mathrm{Exp}(\lambda)$, IID, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
So under $H_0 : \mu = E(X_i) = \frac{1}{2}$
Assume that we decide to accept H_0 if $\bar{X} > 0.45$; calculate α

- $\bar{X} > 0.45 \Leftrightarrow \sum_{i=1}^{5} X_i > 5 \cdot 0.45 = 2.25$
  Let $Y = \sum_{i=1}^{5} X_i$ (sum of independent exponentially distributed RV's)

$\Rightarrow$ $Y \sim \mathrm{Erl}(\lambda = 2, n = 5)$ (under H_0), $E(Y) = 5/2$

- Apparently:

Example 2 (a)

$X_1, \ldots, X_5 \sim \mathrm{Exp}(\lambda)$, IID, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
So under $H_0 : \mu = E(X_i) = \frac{1}{2}$
Assume that we decide to accept H_0 if $\bar{X} > 0.45$; calculate α

- $\bar{X} > 0.45 \Leftrightarrow \sum_{i=1}^{5} X_i > 5 \cdot 0.45 = 2.25$
  Let $Y = \sum_{i=1}^{5} X_i$ (sum of independent exponentially distributed RV's)

$\Rightarrow$ $Y \sim \mathrm{Erl}(\lambda = 2, n = 5)$ (under H_0), $E(Y) = 5/2$

- Apparently: $\alpha = P(Y \leq 2.25)$

Example 2 (a)

$X_1, \ldots, X_5 \sim \mathrm{Exp}(\lambda)$, IID, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
So under $H_0 : \mu = E(X_i) = \frac{1}{2}$
Assume that we decide to accept H_0 if $\bar{X} > 0.45$; calculate α

- $\bar{X} > 0.45 \Leftrightarrow \sum_{i=1}^{5} X_i > 5 \cdot 0.45 = 2.25$
  Let $Y = \sum_{i=1}^{5} X_i$ (sum of independent exponentially distributed RV's)

$\Rightarrow$ $Y \sim \mathrm{Erl}(\lambda = 2, n = 5)$ (under H_0), $E(Y) = 5/2$

- Apparently: $\alpha = P(Y \leq 2.25) = \int_0^{2.25} \frac{\lambda^n x^{n-1}}{(n-1)!} \exp(-\lambda x)\,\mathrm{d}x$

Example 2 (a)

$X_1, \ldots, X_5 \sim \mathrm{Exp}(\lambda)$, IID, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
So under $H_0 : \mu = E(X_i) = \frac{1}{2}$
Assume that we decide to accept H_0 if $\bar{X} > 0.45$; calculate α

- $\bar{X} > 0.45 \Leftrightarrow \sum_{i=1}^{5} X_i > 5 \cdot 0.45 = 2.25$
  Let $Y = \sum_{i=1}^{5} X_i$ (sum of independent exponentially distributed RV's)

$\Rightarrow$ $Y \sim \mathrm{Erl}(\lambda = 2, n = 5)$ (under H_0), $E(Y) = 5/2$

- Apparently: $\alpha = P(Y \leq 2.25) = \int_0^{2.25} \frac{\lambda^n x^{n-1}}{(n-1)!} \exp(-\lambda x)\, \mathrm{d}x = \int_0^{2.25} \frac{2^5 x^4}{4!} \exp(-2x)\, \mathrm{d}x = \frac{4}{3} \int_0^{2.25} x^4 \exp(-2x)\, \mathrm{d}x$

Example 2 (a)

$X_1, \ldots, X_5 \sim \mathrm{Exp}(\lambda)$, IID, λ unknown
$H_0 : \lambda = 2$ vs. $H_1 : \lambda > 2$
So under $H_0 : \mu = E(X_i) = \frac{1}{2}$
Assume that we decide to accept H_0 if $\bar{X} > 0.45$; calculate α

- $\bar{X} > 0.45 \Leftrightarrow \sum_{i=1}^{5} X_i > 5 \cdot 0.45 = 2.25$
  Let $Y = \sum_{i=1}^{5} X_i$ (sum of independent exponentially distributed RV's)

$\Rightarrow$ $Y \sim \mathrm{Erl}(\lambda = 2, n = 5)$ (under H_0), $E(Y) = 5/2$

- Apparently: $\alpha = P(Y \leq 2.25) = \int_0^{2.25} \frac{\lambda^n x^{n-1}}{(n-1)!} \exp(-\lambda x)\, \mathrm{d}x =$
  $\int_0^{2.25} \frac{2^5 x^4}{4!} \exp(-2x)\, \mathrm{d}x = \frac{4}{3} \int_0^{2.25} x^4 \exp(-2x)\, \mathrm{d}x = \ldots \approx 0.4679$

Two-sided tests vs. one-sided tests

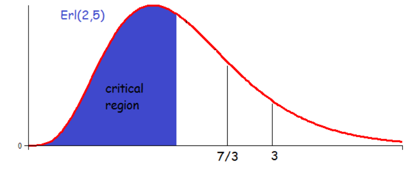
Example 2 (b)

Example 2 (b)

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?

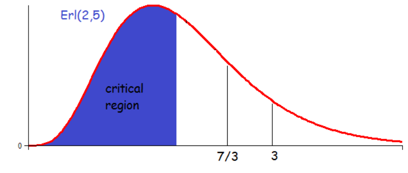
Example 2 (b)

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



Example 2 (b)

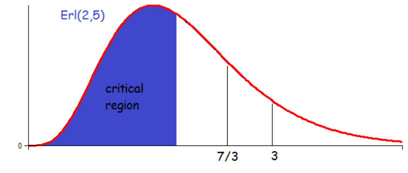
We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(y) = 2\frac{1}{2}$

Example 2 (b)

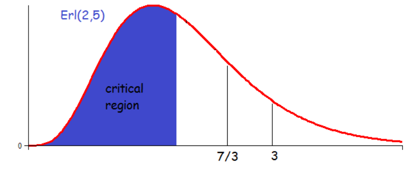
We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(y) = 2\frac{1}{2}$
Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$,

Example 2 (b)

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?

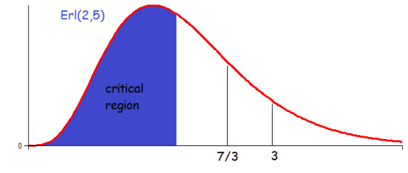


$E(y) = 2\frac{1}{2}$

Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2$

Example 2 (b)

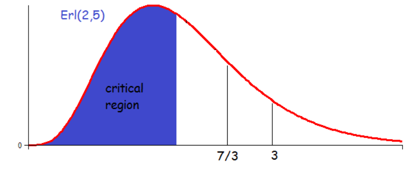
We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(y) = 2\frac{1}{2}$
Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \rightarrow$
Never reject H_0

Example 2 (b)

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(y) = 2\frac{1}{2}$

Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \rightarrow$

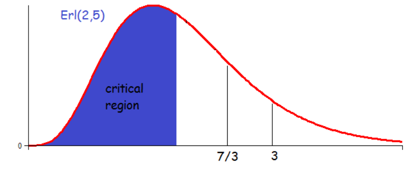
Never reject H_0

Example 2 (c)

Two-sided tests vs. one-sided tests

Example 2 (b)

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



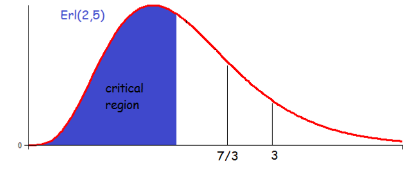
$E(y) = 2\frac{1}{2}$
Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \rightarrow$
Never reject H_0

Example 2 (c)

Suppose $y = \sum_{i=1}^{5} x_i = 2$

Example 2 (b)

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(y) = 2\frac{1}{2}$

Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \rightarrow$

Never reject H_0

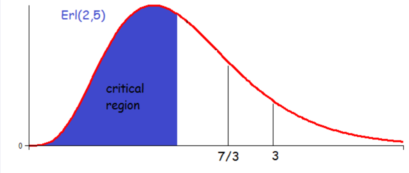
Example 2 (c)

Suppose $y = \sum_{i=1}^{5} x_i = 2 < 2.5$

Two-sided tests vs. one-sided tests

Example 2 (b)

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



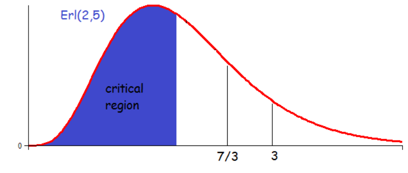
$E(y) = 2\frac{1}{2}$
Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \rightarrow$
Never reject H_0

Example 2 (c)

Suppose $y = \sum_{i=1}^{5} x_i = 2 < 2.5$
For what α do we accept H_0 (i.e., what is the P-value for $y = 2$?)

Example 2 (b)

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(y) = 2\frac{1}{2}$
Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \rightarrow$
Never reject H_0

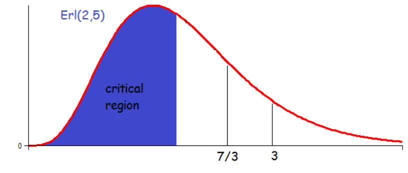
Example 2 (c)

Suppose $y = \sum_{i=1}^{5} x_i = 2 < 2.5$
For what α do we accept H_0 (i.e., what is the P-value for $y = 2$?)

- P-value: $P(Y \leq 2 | H_0$ is true)

Example 2 (b)

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(y) = 2\frac{1}{2}$
Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \rightarrow$
Never reject H_0

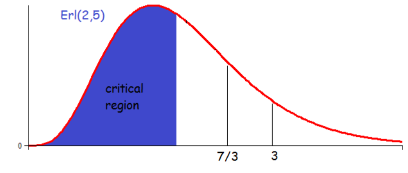
Example 2 (c)

Suppose $y = \sum_{i=1}^{5} x_i = 2 < 2.5$
For what α do we accept H_0 (i.e., what is the P-value for $y = 2$?)

- P-value: $P(Y \leq 2 | H_0 \text{ is true}) = P\left(Y \leq 2 | Y \sim \mathrm{Erl}(\lambda = 2, n = 5)\right)$

Example 2 (b)

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(y) = 2\frac{1}{2}$
Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \rightarrow$
Never reject H_0

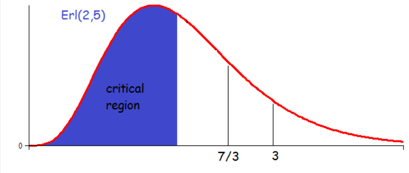
Example 2 (c)

Suppose $y = \sum_{i=1}^{5} x_i = 2 < 2.5$
For what α do we accept H_0 (i.e., what is the P-value for $y = 2$?)

- P-value: $P(Y \leq 2 | H_0 \text{ is true}) = P\left(Y \leq 2 | Y \sim \mathrm{Erl}(\lambda = 2, n = 5)\right)$
  $= \frac{4}{3} \int_0^2 x^4 \exp(-2\,x)\,\mathrm{d}x$

Example 2 (b)

We found $\sum_{i=1}^{5} x_i = 3$. For what levels of significance we accept $H_0 : \lambda \leq 2$?



$E(y) = 2\frac{1}{2}$
Realization: $y = 3 \Rightarrow$ Corresponds to a λ-value of $\frac{5}{3} < 2$, but $H_1 : \lambda > 2 \rightarrow$
Never reject H_0

Example 2 (c)

Suppose $y = \sum_{i=1}^{5} x_i = 2 < 2.5$
For what α do we accept H_0 (i.e., what is the P-value for $y = 2$?)

- P-value: $P(Y \leq 2 | H_0 \text{ is true}) = P\left(Y \leq 2 | Y \sim \mathrm{Erl}(\lambda = 2, n = 5)\right)$
  $= \frac{4}{3} \int_0^2 x^4 \exp(-2\,x)\,\mathrm{d}x \ = \ldots = 1 - 34 \cdot \frac{1}{3} \cdot \exp(-4) = 0.3712$
  (probably accept H_0)