Lecture 2: Concepts of Probability Theory

Kateřina Staňková

Statistics (MAT1003)

April 11, 2012

◆□▶ ◆□▶ ▲□▶ ▲□▶ ■ ののの

◆□▶ ◆□▶ ▲□▶ ▲□▶ ■ ののの

Outline

Concepts from Probability Theory

- Experiment, Sample Space, Event
- Complement of an event, intersection of events, disjoint/mutually exclusive events, union of events

Probabilities

- Probability measure, probability of an event
- Examples
- Calculating probabilities



◆□▶ ◆□▶ ▲□▶ ▲□▶ ■ ののの

And now ...

Concepts from Probability Theory

- Experiment, Sample Space, Event
- Complement of an event, intersection of events, disjoint/mutually exclusive events, union of events

2 Probabilities

- Probability measure, probability of an event
- Examples
- Calculating probabilities



Random variables

Experiment, Sample Space, Event

book: Sections 2.1, 2.2

Experiment

Throw a single die Draw one card out of a deck

Sample Space (assumption: finite & countable)

$S = \{1, 2, 3, 4, 5, 6\}$ S = $\{1, 2, 3, 4, 5, 6\}$

Event

Event *A* : The outcome is a multiple of 3, i.e. A={3,6} *C* : A queen, i.e., $C = \{\clubsuit Q, \diamondsuit Q, \heartsuit Q, \clubsuit Q\}$

Random variables

Experiment, Sample Space, Event

book: Sections 2.1, 2.2

Experiment

Action with the outcome determined by chance

Throw a single die

Draw one card out of a deck

Sample Space (assumption: finite & countable)

 $S = \{1, 2, 3, 4, 5, 6\}$

Event

Event *A* : The outcome is a multiple of 3, i.e. A={3,6} *C* : A queen, i.e., $C = \{\clubsuit Q, \diamondsuit Q, \heartsuit Q, \clubsuit Q\}$

▲□▶▲□▶▲□▶▲□▶ □ つへで

Random variables

book: Sections 2.1, 2.2

Experiment

Action with the outcome determined by chance

Throw a single die

Draw one card out of a deck

Sample Space (assumption: finite & countable)

The set of possible outcomes of an experiment (denoted S)

Event

Event *A* : The outcome is a multiple of 3, i.e. A={3,6} *C* : A queen, i.e., $C = \{ \clubsuit Q, \Diamond Q, \Diamond Q, \spadesuit Q \}$

Random variables

Experiment, Sample Space, Event

book: Sections 2.1, 2.2

Experiment

Action with the outcome determined by chance

Throw a single die

Draw one card out of a deck

Sample Space (assumption: finite & countable)

The set of possible outcomes of an experiment (denoted S)

Event

A subset of the sample space (Note: It is a set!)

 $C : A queen, i.e., C = \{\clubsuit Q, \diamondsuit Q, \heartsuit Q, \blacklozenge Q\}$

Random variables

Experiment, Sample Space, Event

book: Sections 2.1, 2.2

Experiment

Action with the outcome determined by chance Throw a single die

Draw one card out of a deck

Sample Space (assumption: finite & countable)

The set of possible outcomes of an experiment (denoted S)

 $S = \{1, 2, 3, 4, 5, 6\}$

Event

A subset of the sample space (Note: It is a set!) Event A : The outcome is a multiple of 3, i.e. $A=\{3,6\}$ C : A queen, i.e., $C = \{AQ, QQ, PQ, AQ\}$

◆□▶◆圖▶◆臣▶◆臣▶ 臣 の�?

Random variables

Experiment, Sample Space, Event

book: Sections 2.1, 2.2

Experiment

Action with the outcome determined by chance Throw a single die

Draw one card out of a deck

Sample Space (assumption: finite & countable)

The set of possible outcomes of an experiment (denoted S) $S = \{1, 2, 3, 4, 5, 6\}$

Event

A subset of the sample space (Note: It is a set!) Event A : The outcome is a multiple of 3, i.e. $A={3,6}$ C : A queen, i.e., $C = {AQ, QQ, QQ, AQ}$

・ロト・日本 キャー モー シック

Random variables

Experiment, Sample Space, Event

book: Sections 2.1, 2.2

Experiment

Action with the outcome determined by chance Throw a single die

Draw one card out of a deck

Sample Space (assumption: finite & countable)

The set of possible outcomes of an experiment (denoted S) $S = \{1, 2, 3, 4, 5, 6\}$

Event

A subset of the sample space (Note: It is a set!) Event A: The outcome is a multiple of 3, i.e. $A=\{3,6\}$

Random variables

Experiment, Sample Space, Event

book: Sections 2.1, 2.2

Experiment

Action with the outcome determined by chance Throw a single die Draw one card out of a deck

Sample Space (assumption: finite & countable)

The set of possible outcomes of an experiment (denoted *S*) $S = \{1, 2, 3, 4, 5, 6\}$

Event

A subset of the sample space (Note: It is a set!) Event *A* : The outcome is a multiple of 3, i.e. A={3,6} *C* : A queen, i.e., $C = \{ \clubsuit Q, \Diamond Q, \Diamond Q, \spadesuit Q \}$

Random variables

Experiment, Sample Space, Event

book: Sections 2.1, 2.2

Experiment

Action with the outcome determined by chance Throw a single die Draw one card out of a deck

Sample Space (assumption: finite & countable)

Event

A subset of the sample space (Note: It is a set!) Event *A* : The outcome is a multiple of 3, i.e. A={3,6} *C* : A queen, i.e., $C = \{ \clubsuit Q, \Diamond Q, \Diamond Q, \spadesuit Q \}$

Random variables

Experiment, Sample Space, Event

book: Sections 2.1, 2.2

Experiment

Action with the outcome determined by chance Throw a single die Draw one card out of a deck

Sample Space (assumption: finite & countable)

Event

A subset of the sample space (Note: It is a set!) Event *A* : The outcome is a multiple of 3, i.e. A={3,6} *C* : A queen, i.e., $C = \{\clubsuit Q, \diamondsuit Q, \heartsuit Q, \clubsuit Q\}$

Random variables

Experiment, Sample Space, Event

Experiment

Sample Space (assumption: finite & countable)

Event

D : three times the same number, i.e. $D = \{111, 222, 333, 444, 555, 666\}$

Random variables

Experiment, Sample Space, Event

Experiment

Throwing a coin twice

Sample Space (assumption: finite & countable)

Event

D : three times the same number, i.e. $D = \{111, 222, 333, 444, 555, 666\}$

Random variables

Experiment, Sample Space, Event

Experiment

Throwing a coin twice

Sample Space (assumption: finite & countable)

 $S = \{HH, HT, TH, TT\}$ 2 · 2 options

Event

D : three times the same number, i.e. $D = \{111, 222, 333, 444, 555, 666\}$

Experiment, Sample Space, Event

Experiment

Throwing a coin twice

Sample Space (assumption: finite & countable)

 $S = \{HH, HT, TH, TT\}$ 2 · 2 options

Event

Event B : 2 tails, i.e. B={TT}

D : three times the same number, i.e.

 $D = \{111, 222, 333, 444, 555, 666\}$

・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・
・

Experiment, Sample Space, Event

Experiment

Throwing a coin twice

Throwing a die 3 times

Sample Space (assumption: finite & countable)

 $S = \{HH, HT, TH, TT\}$ 2 · 2 options

Event

Event B : 2 tails, i.e. B={TT}

D: three times the same number, i.e. $D = \{111, 222, 333, 444, 555, 666\}$ Experiment, Sample Space, Event

Experiment

Throwing a coin twice

Throwing a die 3 times

Sample Space (assumption: finite & countable)

 $S = \{HH, HT, TH, TT\}$ 2 · 2 options

 $S = \{111, 112, 113, \dots, 666\} - 66 \cdot 6 = 216$ options

Event

Event B : 2 tails, i.e. B={TT}

D: three times the same number, i.e. $D = \{111, 222, 333, 444, 555, 666\}$ Experiment, Sample Space, Event

Experiment

Throwing a coin twice

Throwing a die 3 times

Sample Space (assumption: finite & countable)

 $S = \{HH, HT, TH, TT\}$ 2 · 2 options

 $S = \{111, 112, 113, \dots, 666\} - 66 \cdot 6 = 216$ options

Event

Event B : 2 tails, i.e. B={TT}

D: three times the same number, i.e. $D = \{111, 222, 333, 444, 555, 666\}$

Events are sets, we can therefore talk about:

Complement of an event A: event A'



Example: throwing die $S = \{1, 2, 3, 4, 5, 6\}, A = \{1, 3, 5\}$ $A' = \{2, 4, 6\}$

Events are sets, we can therefore talk about:

Complement of an event A: event A'



Example: throwing die $S = \{1, 2, 3, 4, 5, 6\}, A = \{1, 3, 5\}$ $A' = \{2, 4, 6\}$

Events are sets, we can therefore talk about:

Complement of an event A: event A'



Example: throwing die

$$S = \{1, 2, 3, 4, 5, 6\}, A = \{1, 3, 5\}$$
$$A' = \{2, 4, 6\}$$

Events are sets, we can therefore talk about:

Complement of an event A: event A'



Example: throwing die $S = \{1, 2, 3, 4, 5, 6\}, A = \{1, 3, 5\}$ $A' = \{2, 4, 6\}$

Events are sets, we can therefore talk about:

Complement of an event A: event A'



Example: throwing die $S = \{1, 2, 3, 4, 5, 6\}, A = \{1, 3, 5\}$ $A' = \{2, 4, 6\}$

Events are sets, we can therefore talk about:

Intersection of events $A \cap B$



Example: throwing a die $S = \{1, 2, 3, 4, 5, 6\}, A = \{1, 3, 5\}, B = \{3, 6\}, C = \{4\}$ $A \cap B = \{3\}$ $A \cap C = \emptyset \Rightarrow A$ and C and disjoint (book: "mutually exclusiv

◆□ ▶ ◆□ ▶ ◆ □ ▶ ◆ □ ▶ ◆ □ ● ◆ ○ へ ○

▲□▶ ▲□▶ ▲三▶ ▲三▶ - 三 - のへで

Complement of an event, intersection of events, disjoint/mutually exclusive events, union of events

Events are sets, we can therefore talk about:





Example: throwing a die

 $S = \{1, 2, 3, 4, 5, 6\}, A = \{1, 3, 5\}, B = \{3, 6\}, C = \{4\}$ $A \cap B = \{3\}$ $A \cap C = \emptyset \Rightarrow A$ and C and disjoint (book: "mutually exclusive

Events are sets, we can therefore talk about:

Intersection of events $A \cap B$



Example: throwing a die $S = \{1, 2, 3, 4, 5, 6\}, A = \{1, 3, 5\}, B = \{3, 6\}, C = \{4\}$ $A \cap B = \{3\}$ $A \cap C = \emptyset \Rightarrow A$ and *C* and disjoint (book: "mutually exclusive

・ロト・西ト・西ト・西ト・日・ シック

◆□▶ ◆□▶ ▲□▶ ▲□▶ ■ ののの

Complement of an event, intersection of events, disjoint/mutually exclusive events, union of events

Events are sets, we can therefore talk about:

Intersection of events $A \cap B$



Example: throwing a die $S = \{1, 2, 3, 4, 5, 6\}, A = \{1, 3, 5\}, B = \{3, 6\}, C = \{4\}$ $A \cap B = \{3\}$ $A \cap C = \emptyset \Rightarrow A \text{ and } C \text{ and disjoint (book: "mutually exclusive)}$

◆□▶ ◆□▶ ▲□▶ ▲□▶ ■ ののの

Complement of an event, intersection of events, disjoint/mutually exclusive events, union of events

Events are sets, we can therefore talk about:

Intersection of events $A \cap B$



Example: throwing a die $S = \{1, 2, 3, 4, 5, 6\}, A = \{1, 3, 5\}, B = \{3, 6\}, C = \{4\}$ $A \cap B = \{3\}$ $A \cap C = \emptyset \Rightarrow A$ and *C* and disjoint (book: "mutually exclusive")

Events are sets, we can therefore talk about:

Union of events: A U B



Example: throwing a die $S = \{1, 2, 3, 4, 5, 6\}, A = \{1, 3, 5\}, B = \{3, 6\}, C = \{4\}$ $A \cup B = \{1, 3, 5, 6\}$

◆□▶ ◆□▶ ◆ □▶ ◆ □ ◆ ○ ◆ ○ ◆ ○ ◆

Events are sets, we can therefore talk about:

Union of events: A U B



Example: throwing a die $S = \{1, 2, 3, 4, 5, 6\}, A = \{1, 3, 5\}, B = \{3, 6\}, C = \{4\}$ $A \cup B = \{1, 3, 5, 6\}$

◆□▶ ◆□▶ ◆ □▶ ◆ □ ◆ ○ ◆ ○ ◆ ○ ◆

Events are sets, we can therefore talk about:

Union of events: A U B



Example: throwing a die $S = \{1, 2, 3, 4, 5, 6\}, A = \{1, 3, 5\}, B = \{3, 6\}, C = \{4\}$ $A \cup B = \{1, 3, 5, 6\}$

◆□▶ ◆□▶ ◆豆▶ ◆豆▶ □ のへで

Events are sets, we can therefore talk about:

Union of events: A U B



Example: throwing a die $S = \{1, 2, 3, 4, 5, 6\}, A = \{1, 3, 5\}, B = \{3, 6\}, C = \{4\}$ $A \cup B = \{1, 3, 5, 6\}$

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● ● ● ●

▲□▶ ▲□▶ ▲ 三▶ ▲ 三▶ - 三 - のへぐ

Complement of an event, intersection of events, disjoint/mutually exclusive events, union of events



◆□▶ ◆□▶ ▲□▶ ▲□▶ ■ ののの

And now ...

Concepts from Probability Theory

- Experiment, Sample Space, Event
- Complement of an event, intersection of events, disjoint/mutually exclusive events, union of events

Probabilities

- Probability measure, probability of an event
- Examples
- Calculating probabilities


・ロン ・ 雪 と ・ ヨ と ・ ヨ ・

Probability measure, probability of an event

book: Section 2.4

Probability measure

A probability measure is a function that assigns to each element of S a number in [0,1] (the probability), such that the sum of these probabilities is 1

Probability of an event

The probability of event A is the sum of the probabilities of the elements in A (notation: P(A))

 $P(S) = 1, P(\emptyset) = 0, 0 \le P(A) \le 1$ for each event A

Probability measure, probability of an event

book: Section 2.4

Probability measure

A probability measure is a function that assigns to each element of S a number in [0,1] (the probability), such that the sum of these probabilities is 1

Probability of an event

The probability of event A is the sum of the probabilities of the elements in A (notation: P(A))

 $P(S) = 1, P(\emptyset) = 0, 0 \le P(A) \le 1$ for each event A

・ロト・西ト・西ト・西ト・日・

(日) (日) (日) (日) (日) (日) (日)

Probability measure, probability of an event

book: Section 2.4

Probability measure

A probability measure is a function that assigns to each element of S a number in [0,1] (the probability), such that the sum of these probabilities is 1

Probability of an event

The probability of event A is the sum of the probabilities of the elements in A (notation: P(A))

$P(S) = 1, P(\emptyset) = 0, 0 \le P(A) \le 1$ for each event A

◆□▶ ◆□▶ ▲□▶ ▲□▶ ■ ののの

Probability measure, probability of an event

book: Section 2.4

Probability measure

A probability measure is a function that assigns to each element of S a number in [0,1] (the probability), such that the sum of these probabilities is 1

Probability of an event

The probability of event A is the sum of the probabilities of the elements in A (notation: P(A))

 $P(S) = 1, P(\emptyset) = 0, 0 \le P(A) \le 1$ for each event A

Fair die

$$S = \{1, 2, 3, 4, 5, 6\}, P(\{1\}) = P(\{2\}) = \ldots = P(\{6\}) = \frac{1}{6}$$

Unfair die

$$S = \{1, 2, 3, 4, 5, 6\}, P(\{1\}) = P(\{2\}) = \ldots = P(\{5\}) = \frac{1}{10}, P(\{6\}) = \frac{1}{2}$$

5 apples, 4 bananas

$$S = \{A_1, A_2, A_3, A_4, A_5, B_1, B_2, B_3, B_4\}, P(\{A_1\}) = \dots = P(\{B_4\}) = \frac{1}{9} P(\{apple\}) = \frac{5}{9}, P(\{banana\}) = \frac{4}{9}$$

◆□ ▶ ◆□ ▶ ◆三 ▶ ◆□ ▶ ◆□ ●

Fair die

$$S = \{1, 2, 3, 4, 5, 6\}, P(\{1\}) = P(\{2\}) = \ldots = P(\{6\}) = \frac{1}{6}$$

Unfair die

$$S = \{1, 2, 3, 4, 5, 6\}, P(\{1\}) = P(\{2\}) = \dots = P(\{5\}) = \frac{1}{10}, P(\{6\}) = \frac{1}{2}$$

5 apples, 4 bananas

$$S = \{A_1, A_2, A_3, A_4, A_5, B_1, B_2, B_3, B_4\},\$$

$$P(\{A_1\}) = \dots = P(\{B_4\}) = \frac{1}{9}$$

$$P(\{apple\}) = \frac{5}{9}, P(\{banana\}) = \frac{4}{9}$$

◆□▶ ◆□▶ ◆∃▶ ◆∃▶ = のへで

Fair die

$$S = \{1, 2, 3, 4, 5, 6\}, P(\{1\}) = P(\{2\}) = \ldots = P(\{6\}) = \frac{1}{6}$$

Unfair die

$$S = \{1, 2, 3, 4, 5, 6\}, P(\{1\}) = P(\{2\}) = \ldots = P(\{5\}) = \frac{1}{10}, P(\{6\}) = \frac{1}{2}$$

5 apples, 4 bananas

$$S = \{A_1, A_2, A_3, A_4, A_5, B_1, B_2, B_3, B_4\},\$$

$$P(\{A_1\}) = \dots = P(\{B_4\}) = \frac{1}{9}$$

$$P(\{apple\}) = \frac{5}{9}, P(\{banana\}) = \frac{4}{9}$$

◆□▶ ◆□▶ ◆∃▶ ◆∃▶ = のへで

Fair die

$$S = \{1, 2, 3, 4, 5, 6\}, P(\{1\}) = P(\{2\}) = \ldots = P(\{6\}) = \frac{1}{6}$$

Unfair die

$$S = \{1, 2, 3, 4, 5, 6\}, P(\{1\}) = P(\{2\}) = \ldots = P(\{5\}) = \frac{1}{10}, P(\{6\}) = \frac{1}{2}$$

5 apples, 4 bananas

$$S = \{A_1, A_2, A_3, A_4, A_5, B_1, B_2, B_3, B_4\}, P(\{A_1\}) = \dots = P(\{B_4\}) = \frac{1}{9} P(\{apple\}) = \frac{5}{9}, P(\{banana\}) = \frac{4}{9}$$

◆□▶ ◆□▶ ◆ □ ▶ ◆ □ ▶ ◆ □ ● ● ● ●

Fair die

$$S = \{1, 2, 3, 4, 5, 6\}, P(\{1\}) = P(\{2\}) = \ldots = P(\{6\}) = \frac{1}{6}$$

Unfair die

$$S = \{1, 2, 3, 4, 5, 6\}, P(\{1\}) = P(\{2\}) = \ldots = P(\{5\}) = \frac{1}{10}, P(\{6\}) = \frac{1}{2}$$

5 apples, 4 bananas

$$S = \{A_1, A_2, A_3, A_4, A_5, B_1, B_2, B_3, B_4\},\$$

$$P(\{A_1\}) = \dots = P(\{B_4\}) = \frac{1}{9}$$

$$P(\{apple\}) = \frac{5}{9}, P(\{banana\}) = \frac{4}{9}$$

◆□▶ ◆□▶ ◆ □ ▶ ◆ □ ▶ ◆ □ ● ● ● ●

5 apples, 4 bananas

- Select 2 pieces, what is the probability of selecting one apple and one banana?
 - $S = \{A_1, A_2\}, \{A_1, A_3\}, \dots, \{B_3, B_4\}$
- selection of 1 apple: 5 ways, selection of 1 banana: 4 ways
- selection of 1 apple and 1 banana: 5 · 4 = 20 ways
- $2: \begin{pmatrix} 9\\ 2 \end{pmatrix} = \sqrt[n]{3} = 20$
- $P(1 \text{ apple and } 1 \text{ banana}) = \frac{9}{6} = \frac{6}{6} = \frac{6}{6}$ • P(2 apples), P(2 bananas) = 7

5 apples, 4 bananas

- Select 2 pieces, what is the probability of selecting one apple and one banana?
- $S = \{A_1, A_2\}, \{A_1, A_3\}, \dots, \{B_3, B_4\}$
- selection of 1 apple: 5 ways, selection of 1 banana: 4 ways
- selection of 1 apple and 1 banana: $5 \cdot 4 = 20$ ways
- total number of selecting 2 pieces combination 9 choose 2 : $\begin{pmatrix} 9 \\ 2 \end{pmatrix} = \frac{9!}{7! \cdot 2!} = 36$

• $P(1 \text{ apple and } 1 \text{ banana}) = \frac{20}{36} = \frac{5}{9} = \frac{|A|}{|S|}$

• P(2 apples), P(2 bananas) =?

5 apples, 4 bananas

- Select 2 pieces, what is the probability of selecting one apple and one banana?
- $S = \{A_1, A_2\}, \{A_1, A_3\}, \dots, \{B_3, B_4\}$
- selection of 1 apple: 5 ways, selection of 1 banana: 4 ways
- selection of 1 apple and 1 banana: $5 \cdot 4 = 20$ ways
- total number of selecting 2 pieces combination 9 choose 2 : $\begin{pmatrix} 9 \\ 2 \end{pmatrix} = \frac{9!}{7! \cdot 2!} = 36$

• $P(1 \text{ apple and } 1 \text{ banana}) = \frac{20}{36} = \frac{5}{9} = \frac{|A|}{|S|}$

• P(2 apples), P(2 bananas) =?

5 apples, 4 bananas

- Select 2 pieces, what is the probability of selecting one apple and one banana?
- $S = \{A_1, A_2\}, \{A_1, A_3\}, \dots, \{B_3, B_4\}$
- selection of 1 apple: 5 ways, selection of 1 banana: 4 ways
- selection of 1 apple and 1 banana: $5 \cdot 4 = 20$ ways
- total number of selecting 2 pieces combination 9 choose 2 : $\begin{pmatrix} 9 \\ 2 \end{pmatrix} = \frac{9!}{7! \cdot 2!} = 36$

• $P(1 \text{ apple and } 1 \text{ banana}) = \frac{20}{36} = \frac{5}{9} = \frac{|A|}{|S|}$

• P(2 apples), P(2 bananas) =?

5 apples, 4 bananas

- Select 2 pieces, what is the probability of selecting one apple and one banana?
- $S = \{A_1, A_2\}, \{A_1, A_3\}, \dots, \{B_3, B_4\}$
- selection of 1 apple: 5 ways, selection of 1 banana: 4 ways
- selection of 1 apple and 1 banana: $5 \cdot 4 = 20$ ways
- total number of selecting 2 pieces combination 9 choose 2 : $\begin{pmatrix} 9 \\ 2 \end{pmatrix} = \frac{9!}{7! \cdot 2!} = 36$

• $P(1 \text{ apple and } 1 \text{ banana}) = \frac{20}{36} = \frac{5}{9} = \frac{|A|}{|S|}$

P(2 apples), P(2 bananas) =?

5 apples, 4 bananas

- Select 2 pieces, what is the probability of selecting one apple and one banana?
- $S = \{A_1, A_2\}, \{A_1, A_3\}, \dots, \{B_3, B_4\}$
- selection of 1 apple: 5 ways, selection of 1 banana: 4 ways
- selection of 1 apple and 1 banana: $5 \cdot 4 = 20$ ways
- total number of selecting 2 pieces combination 9 choose $2: \begin{pmatrix} 9\\2 \end{pmatrix} = \frac{9!}{7! \cdot 2!} = \frac{36}{7! \cdot 2!}$
- P(1 apple and 1 banana) = ²⁰/₃₆ = ⁵/₉ = ^{|A}/_{|S}
 P(2 apples), P(2 bananas) =?

Examples

5 apples, 4 bananas

- Select 2 pieces, what is the probability of selecting one apple and one banana?
- $S = \{A_1, A_2\}, \{A_1, A_3\}, \dots, \{B_3, B_4\}$
- selection of 1 apple: 5 ways, selection of 1 banana: 4 ways
- selection of 1 apple and 1 banana: $5 \cdot 4 = 20$ ways
- total number of selecting 2 pieces combination 9 choose 2 : $\begin{pmatrix} 9 \\ 2 \end{pmatrix} = \frac{9!}{7! \cdot 2!} = \frac{36}{7!}$

• $P(1 \text{ apple and } 1 \text{ banana}) = \frac{20}{36} = \frac{5}{9} = \frac{|A|}{|S|}$

5 apples, 4 bananas

- Select 2 pieces, what is the probability of selecting one apple and one banana?
- $S = \{A_1, A_2\}, \{A_1, A_3\}, \dots, \{B_3, B_4\}$
- selection of 1 apple: 5 ways, selection of 1 banana: 4 ways
- selection of 1 apple and 1 banana: $5 \cdot 4 = 20$ ways
- total number of selecting 2 pieces combination 9 choose $2: \begin{pmatrix} 9\\ 2 \end{pmatrix} = \frac{9!}{7! \cdot 2!} = 36$
- $P(1 \text{ apple and } 1 \text{ banana}) = \frac{20}{36} = \frac{5}{9} = \frac{|A|}{|S|}$

P(2 apples), P(2 bananas) =?

The rules

- $P(A) = \frac{|A|}{|S|}$ if each element of *S* has the same probability (Thm. 2.9)
- For any events A and B: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (Thm. 2.10)
- For disjoint events A and B: P(A ∪ B) = P(A) + P(B) (Thm. 2.10)
- For any event A: P(A) = 1 P(A') (Thm. 2.12) *Proof:* $1 = P(S) = P(A \cup A') = P(A) + P(A')$

・ロト・西ト・西ト・西ト・日・ シック

The rules

- $P(A) = \frac{|A|}{|S|}$ if each element of *S* has the same probability (Thm. 2.9)
- For any events A and B:
 P(*A* ∪ *B*) = *P*(*A*) + *P*(*B*) *P*(*A* ∩ *B*) (Thm. 2.10)
- For disjoint events A and B: P(A ∪ B) = P(A) + P(B) (Thm. 2.10)
- For any event A: P(A) = 1 P(A') (Thm. 2.12) *Proof:* $1 = P(S) = P(A \cup A') = P(A) + P(A')$

The rules

- $P(A) = \frac{|A|}{|S|}$ if each element of *S* has the same probability (Thm. 2.9)
- For any events A and B: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (Thm. 2.10)
- For disjoint events A and B: *P*(*A* ∪ *B*) = *P*(*A*) + *P*(*B*) (Thm. 2.10)
- For any event A: P(A) = 1 P(A') (Thm. 2.12) *Proof:* $1 = P(S) = P(A \cup A') = P(A) + P(A')$

The rules

- $P(A) = \frac{|A|}{|S|}$ if each element of *S* has the same probability (Thm. 2.9)
- For any events A and B: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (Thm. 2.10)
- For disjoint events A and B: *P*(*A* ∪ *B*) = *P*(*A*) + *P*(*B*) (Thm. 2.10)
- For any event A: P(A) = 1 P(A') (Thm. 2.12) *Proof:* $1 = P(S) = P(A \cup A') = P(A) + P(A')$

Calculating probabilities

Example: Throw a die

$$S = \{1, 2, 3, 4, 5, 6\}, \text{ events: } A = \{1\}, B = \{2, 4, 6\}, \\ C = \{1, 2, 3\} \\ P(A) = ?, P(B \cup C) = ?, P(A \cup B) = ?, P(C') = ? \\ P(A) = \frac{|A|}{|S|} = \frac{1}{6} \\ P(B \cup C) = P(B) + P(C) - P(B \cap C) = \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \frac{5}{6} \\ P(A \cup B) = P(A) + P(B) = \frac{4}{6} \\ P(C') = 1 - P(C)$$

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 臣 のへぐ

Calculating probabilities

Example: Throw a die

$$S = \{1, 2, 3, 4, 5, 6\}, \text{ events: } A = \{1\}, B = \{2, 4, 6\}, \\ C = \{1, 2, 3\} \\ P(A) = ?, P(B \cup C) = ?, P(A \cup B) = ?, P(C') = ? \\ P(A) = \frac{|A|}{|S|} = \frac{1}{6} \\ P(B \cup C) = P(B) + P(C) - P(B \cap C) = \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \frac{5}{6} \\ P(A \cup B) = P(A) + P(B) = \frac{4}{6} \\ P(C') = 1 - P(C)$$

◆□ ▶ ◆□ ▶ ◆ □ ▶ ◆ □ ▶ ● □ ● ● ● ●

▲□▶ ▲□▶ ▲□▶ ▲□▶ ▲□ ● のへぐ

Calculating probabilities

Example: Throw a die

$$S = \{1, 2, 3, 4, 5, 6\}, \text{ events: } A = \{1\}, B = \{2, 4, 6\}, \\ C = \{1, 2, 3\} \\ P(A) = ?, P(B \cup C) = ?, P(A \cup B) = ?, P(C') = ? \\ P(A) = \frac{|A|}{|S|} = \frac{1}{6} \\ P(B \cup C) = P(B) + P(C) - P(B \cap C) = \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \frac{5}{6} \\ P(A \cup B) = P(A) + P(B) = \frac{4}{6} \\ P(C') = 1 - P(C)$$

Calculating probabilities

Example: Throw a die

$$S = \{1, 2, 3, 4, 5, 6\}, \text{ events: } A = \{1\}, B = \{2, 4, 6\}, \\ C = \{1, 2, 3\} \\ P(A) = ?, P(B \cup C) = ?, P(A \cup B) = ?, P(C') = ? \\ P(A) = \frac{|A|}{|S|} = \frac{1}{6} \\ P(B \cup C) = P(B) + P(C) - P(B \cap C) = \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \frac{5}{6} \\ P(A \cup B) = P(A) + P(B) = \frac{4}{6} \\ P(C') = 1 - P(C)$$

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 臣 のへぐ

Calculating probabilities

Example: Throw a die

$$S = \{1, 2, 3, 4, 5, 6\}, \text{ events: } A = \{1\}, B = \{2, 4, 6\},\$$

$$C = \{1, 2, 3\}$$

$$P(A) = ?, P(B \cup C) = ?, P(A \cup B) = ?, P(C') = ?$$

$$P(A) = \frac{|A|}{|S|} = \frac{1}{6}$$

$$P(B \cup C) = P(B) + P(C) - P(B \cap C) = \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \frac{5}{6}$$

$$P(A \cup B) = P(A) + P(B) = \frac{4}{6}$$

$$P(C') = 1 - P(C)$$

◆□ ▶ ◆□ ▶ ◆ □ ▶ ◆ □ ▶ ● □ ● ● ● ●

And now ...

Concepts from Probability Theory

- Experiment, Sample Space, Event
- Complement of an event, intersection of events, disjoint/mutually exclusive events, union of events

2 Probabilities

- Probability measure, probability of an event
- Examples
- Calculating probabilities



Book: Section 3.1

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Example [·]

 $S = \{1, 2, 3, 4\}$ X :square: $\forall s \in S : X(s) = s^2$ Y : 2s if s is odd, $\frac{s}{2}$ if s is even Y(1) = 2, Y(2) = 1, Y(3) = 6, Y(4) = 2

Example 2

Book: Section 3.1

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Example 1

$$S = \{1, 2, 3, 4\}$$

X :square: $\forall s \in S : X(s) = s^2$
Y : 2 s if s is odd, $\frac{s}{2}$ if s is even
 $Y(1) = 2, Y(2) = 1, Y(3) = 6, Y(4) = 2$

Example 2

Book: Section 3.1

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Example 1

$$S = \{1, 2, 3, 4\}$$

X :square: $\forall s \in S : X(s) = s^2$
Y : 2 s if s is odd, $\frac{s}{2}$ if s is even
 $Y(1) = 2, Y(2) = 1, Y(3) = 6, Y(4) = 2$

Example 2

Book: Section 3.1

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Example 1

$$S = \{1, 2, 3, 4\}$$

X :square: $\forall s \in S : X(s) = s^2$
Y : 2 s if s is odd, $\frac{5}{2}$ if s is even
Y(1) = 2, Y(2) = 1, Y(3) = 6, Y(4) = 1

Example 2

Random variables

Book: Section 3.1

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Example 1

$$S = \{1, 2, 3, 4\}$$

X :square: $\forall s \in S : X(s) = s^2$
Y : 2 s if s is odd, $\frac{s}{2}$ if s is even
 $Y(1) = 2, Y(2) = 1, Y(3) = 6, Y(4) = 2$

Example 2

Book: Section 3.1

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Example 1

$$S = \{1, 2, 3, 4\}$$

X :square: $\forall s \in S : X(s) = s^2$
Y : 2 s if s is odd, $\frac{s}{2}$ if s is even
 $Y(1) = 2, Y(2) = 1, Y(3) = 6, Y(4) = 2$

Example 2

Book: Section 3.1

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Example 1

$$S = \{1, 2, 3, 4\}$$

X :square: $\forall s \in S : X(s) = s^2$
Y : 2 s if s is odd, $\frac{s}{2}$ if s is even
 $Y(1) = 2, Y(2) = 1, Y(3) = 6, Y(4) = 2$

Example 2

Book: Section 3.1

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Example 1

$$S = \{1, 2, 3, 4\}$$

X :square: $\forall s \in S : X(s) = s^2$
Y : 2 s if s is odd, $\frac{s}{2}$ if s is even
 $Y(1) = 2, Y(2) = 1, Y(3) = 6, Y(4) = 2$

Example 2

Throwing a die until a 6 comes up

 $S = \{6, N6, NN6, NN6, \dots\}$ X : number of throws required $X(6) = 1, X(N6) = 2, X(NN6) = 3, \dots$

Book: Section 3.1

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Example 1

$$S = \{1, 2, 3, 4\}$$

X :square: $\forall s \in S : X(s) = s^2$
Y : 2 s if s is odd, $\frac{s}{2}$ if s is even
 $Y(1) = 2, Y(2) = 1, Y(3) = 6, Y(4) = 2$

Example 2

Throwing a die until a 6 comes up $S = \{6, N6, NN6, NN6, \ldots\}$ X: number of throws required $X(6) = 1, X(N6) = 2, X(NN6) = 3, \ldots$
Random variables

Book: Section 3.1

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Example 1

$$S = \{1, 2, 3, 4\}$$

X :square: $\forall s \in S : X(s) = s^2$
Y : 2 s if s is odd, $\frac{s}{2}$ if s is even
 $Y(1) = 2, Y(2) = 1, Y(3) = 6, Y(4) = 2$

Example 2

Throwing a die until a 6 comes up $S = \{6, N6, NN6, NN6, ...\}$ X : number of throws required X(6) = 1, X(N6) = 2, X(NN6) = 3,

Book: Section 3.1

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Example 1

$$S = \{1, 2, 3, 4\}$$

X :square: $\forall s \in S : X(s) = s^2$
Y : 2 s if s is odd, $\frac{s}{2}$ if s is even
 $Y(1) = 2, Y(2) = 1, Y(3) = 6, Y(4) = 2$

Example 2

Throwing a die until a 6 comes up $S = \{6, N6, NN6, NN6, \ldots\}$ X: number of throws required $X(6) = 1, X(N6) = 2, X(NN6) = 3, \ldots$

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Discrete vs. continuous random variable

X is a discrete if its set of possible outcomes is finite and countable Otherwise *X* is continuous

Example

S = [1, 2] $X(s) = \frac{1}{s} \forall s \in S \Rightarrow X \in [\frac{1}{2}, 1] \text{ is continuous}$ $Y(s) = \lfloor 2s \rfloor \forall s \in S \Rightarrow Y(s) = \begin{cases} 2, & \text{if } s \in [3,] \\ 3, & \text{if } s \in [4,] \\ 4, & \text{if } s = [2,] \end{cases}$

000

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Discrete vs. continuous random variable

X is a discrete if its set of possible outcomes is finite and countable Otherwise X is continuous

$$S = [1, 2]$$

$$X(s) = \frac{1}{s} \forall s \in S \Rightarrow X \in [\frac{1}{2}, 1] \text{ is continuous}$$

$$Y(s) = \lfloor 2s \rfloor \forall s \in S \Rightarrow Y(s) = \begin{cases} 2, & \text{if } s \in [\frac{1}{2}, \frac{1}{2}, \frac{$$

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Discrete vs. continuous random variable

X is a discrete if its set of possible outcomes is finite and countable

Otherwise X is continuous

Example

$$S = [1, 2]$$

$$X(s) = \frac{1}{s} \forall s \in S \Rightarrow X \in [\frac{1}{2}, 1] \text{ is continuous}$$

$$(2, \text{ if } s \in S)$$

 $Y(s) = \lfloor 2 s
floor \ orall s \in S \Rightarrow Y(s) = 0$

 \Rightarrow Y \in {2,3,4} is discrete

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Discrete vs. continuous random variable

X is a discrete if its set of possible outcomes is finite and countable Otherwise X is continuous

Example

S = [1,2] $X(s) = \frac{1}{s} \forall s \in S \Rightarrow X \in [\frac{1}{2}, 1] \text{ is continuous}$ $Y(s) = \lfloor 2s \rfloor \forall s \in S \Rightarrow Y(s) = \begin{cases} 2, & \text{if } s \in [1, \frac{1}{2}, \frac{1}{2}] \\ 3, & \text{if } s \in [1, \frac{1}{2}, \frac{1}{2}] \\ 4, & \text{if } s = 2 \end{cases}$ $\Rightarrow Y \in \{2, 3, 4\} \text{ is discrete}$

Random variable

X is a random variable for the sample space *S* if it assigns a real number to each element of *S*, *X* : *S* $\rightarrow \mathbb{R}$

Discrete vs. continuous random variable

X is a discrete if its set of possible outcomes is finite and countable Otherwise X is continuous

$$S = [1, 2]$$

$$X(s) = \frac{1}{s} \forall s \in S \Rightarrow X \in [\frac{1}{2}, 1] \text{ is continuous}$$

$$Y(s) = \lfloor 2s \rfloor \forall s \in S \Rightarrow Y(s) = \begin{cases} 2, & \text{if } s \in [1, 1\frac{1}{2}) \\ 3, & \text{if } s \in [1\frac{1}{2}, 2) \\ 4, & \text{if } s = 2 \end{cases}$$

$$\Rightarrow Y \in \{2, 3, 4\} \text{ is discrete}$$

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Discrete vs. continuous random variable

X is a discrete if its set of possible outcomes is finite and countable Otherwise X is continuous

Example

S = [1, 2] $X(s) = \frac{1}{s} \forall s \in S \Rightarrow X \in [\frac{1}{2}, 1] \text{ is continuous}$ $Y(s) = \lfloor 2s \rfloor \forall s \in S \Rightarrow Y(s) = \begin{cases} 2, & \text{if } s \in [1, 1] \\ 3, & \text{if } s \in [1, \frac{1}{2}] \\ 4, & \text{if } s = 2 \end{cases}$ $\Rightarrow Y \in \{2, 3, 4\} \text{ is discrete}$

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Discrete vs. continuous random variable

X is a discrete if its set of possible outcomes is finite and countable Otherwise X is continuous

Example

S = [1, 2] $X(s) = \frac{1}{s} \forall s \in S \Rightarrow X \in [\frac{1}{2}, 1] \text{ is continuous}$ $Y(s) = [2s] \forall s \in S \Rightarrow Y(s) = \begin{cases} 2, & \text{if } s \in [1] \\ 3, & \text{if } s \in [1] \\ 4, & \text{if } s = 2 \end{cases}$

a a

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Discrete vs. continuous random variable

X is a discrete if its set of possible outcomes is finite and countable Otherwise X is continuous

$$S = [1, 2]$$

$$X(s) = \frac{1}{s} \forall s \in S \Rightarrow X \in [\frac{1}{2}, 1] \text{ is continuous}$$

$$Y(s) = \lfloor 2s \rfloor \forall s \in S \Rightarrow Y(s) = \begin{cases} 2, & \text{if } s \in [1] \\ 3, & \text{if } s \in [1] \\ 4, & \text{if } s = 2 \end{cases}$$

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Discrete vs. continuous random variable

X is a discrete if its set of possible outcomes is finite and countable Otherwise X is continuous

Example

$$S = [1, 2]$$

$$X(s) = \frac{1}{s} \forall s \in S \Rightarrow X \in [\frac{1}{2}, 1] \text{ is continuous}$$

$$Y(s) = \lfloor 2s \rfloor \ \forall s \in S \Rightarrow Y(s) = \begin{cases} 2, & \text{if } s \in [1] \\ 3, & \text{if } s \in [1] \\ 4, & \text{if } s = 2 \end{cases}$$

 \Rightarrow Y \in {2,3,4} is discrete

Random variable

X is a random variable for the sample space S if it assigns a real number to each element of $S, X : S \to \mathbb{R}$

Discrete vs. continuous random variable

X is a discrete if its set of possible outcomes is finite and countable Otherwise X is continuous

$$S = [1, 2]$$

$$X(s) = \frac{1}{s} \forall s \in S \Rightarrow X \in [\frac{1}{2}, 1] \text{ is continuous}$$

$$Y(s) = \lfloor 2s \rfloor \forall s \in S \Rightarrow Y(s) = \begin{cases} 2, & \text{if } s \in [1, 1\frac{1}{2}, 2] \\ 3, & \text{if } s \in [1\frac{1}{2}, 2] \\ 4, & \text{if } s = 2 \end{cases}$$

$$\Rightarrow Y \in \{2, 3, 4\} \text{ is discrete}$$