Lecture 3: Probabilities and distributions (Part 1)

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Statistics (MAT1003)

April 16, 2012

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Repetition: Random variables

2 Discrete Probability Distribution

3 Continuous PD



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And now ...



2 Discrete Probability Distribution

3 Continuous PD

Homework - bonus exercises

Random variable

X is a random variable for the sample space *S* if it assigns a real number to each element of *S*, *X* : *S* $\rightarrow \mathbb{R}$

Example

 $S = \{1, 2, 3, 4\}$ X :square: $\forall s \in S : X(s) = s^2$ Y : 2 · s if s is odd, $\frac{s}{2}$ if s is even Y(1) = 2, Y(2) = 1, Y(3) = 6, Y(4) = 2

Example 2

Throwing a die until a 6 comes up $S = \{6, N, 6, N, N, 6, N, N, 8, ...\}$ X: number of throws required X(6) = 1, X(N6) = 2, X(NN6) = 3, ...

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Discrete vs. continuous random variable

X is a discrete if its set of possible outcomes is finite and countable Otherwise *X* is continuous

Example

$$S = [1, 2]$$

$$X(s) = \frac{1}{s} \forall s \in S \Rightarrow X \in [\frac{1}{2}, 1] \text{ is continuous}$$

$$Y(s) = \lfloor 2s \rfloor \forall s \in S \Rightarrow Y(s) = \begin{cases} 2, & \text{if } s \in \\ 3, & \text{if } s \in \\ 4, & \text{if } s = \end{cases}$$

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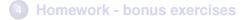
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2 Discrete Probability Distribution

3 Continuous PD





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book: Section 3.2, recommended exercises: 3.11, 3.13, 3.15

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Cumulative distribution F(x)

$$P(X \le x) = F(x) = \sum_{y \le x} f(y)$$

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Exercise - from Example 3.6

A shipment of 20 similar laptop computers to a retail outlet contains 3 that are defective. If a school makes a random purchase of 2 of these computers

- (a) find the probability distribution for the number of deffectives.
- (b) find the cumulative distribution function F(x)

Exercise

Let X be a random variable giving the number of heads minus the number of tails in three tosses of a coin. Find

- (a) the probability distribution of this random variable
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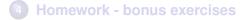
- (a) the probability distribution of this random variable
- (b) find the cumulative distribution function F(x)

And now ...



2 Discrete Probability Distribution

3 Continuous PD



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book: Section 3.3, recommended exercises: 3.9, 3.17, 3.29, 3.31, 3.33

Example

S = [0, 1], X(s) = s, and suppose that all outcomes are "equally likely".

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Observation

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Cumulative distribution

$$F(x) = P(X \le x)$$
 - well defined this way

For continuous RV defined as f(x) = F'(x)

Example



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For continuous RV defined as f(x) = F'(x)

Example

$$F(x) = \begin{cases} 0, & \text{if } x \le 0, \\ x, & \text{if } 0 \le x \le 1, \\ 1, & \text{if } x \ge 1 \end{cases}$$

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2 Discrete Probability Distribution

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Bring to the lecture on April 23

We have a density function of an random variable X, defined as

$$f(x) = \begin{cases} \frac{7-2x}{2}, & 2 < x < 3, \\ 0, & \text{else} \end{cases}$$

Calculate:

(a)
$$P(x \le 2\frac{1}{2})$$

(b) $F(x)$

- 2 Book (pp. 91-92): Exercises 3.6, 3.24
- Book (pp. 70-71): Exercises 2.80, 2.84
- Book (pp. 104-105): Exercises 3.40, 3.44