Lecture 8: Revision, Expectation

Kateřina Staňková

Statistics (MAT1003)

May 1, 2012

Outline

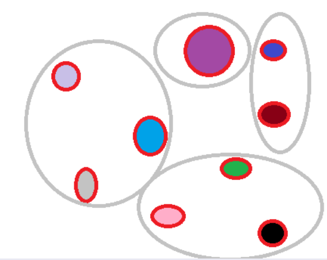
And now . . .

1. **What Did We Do Last Time?**
   - Partition of a sample space
   - Bayes′ rule
2. **Expectation (Mean)**
   - Expectation (Mean) of Discrete Random Variable
   - Expectation (Mean) of Continuous Random Variable
3. **Expectations (Means) of Functions of RVs**
4. **Exercises**

Partition of a sample space

Let S be a sample space. A set of subsets $B_1, B_2, \ldots, B_k$ of S is called a partition of S if

- $\bigcup_{i=1}^{k} B_i = B_1 \cup B_2 \cup \ldots B_k = S$
- $B_i \cap B_j = \emptyset$ for all $i, j \in \{1, \ldots, k\}, i \neq j$



Bayes$'$ rule

Bayes$'$ rule

Bayes′ rule

If $P(B_i)$ and $P(A|B_i)$ are given for all i, we can calculate $P(B_i|A)$ as follows:

Bayes′ rule

If $P(B_i)$ and $P(A|B_i)$ are given for all i, we can calculate $P(B_i|A)$ as follows:

$$P(B_i|A) = \frac{P(B_i \cap A)}{P(A)} = \frac{P(B_i) \cdot P(A|B_i)}{P(A)}$$

Bayes′ rule

If $P(B_i)$ and $P(A|B_i)$ are given for all i, we can calculate $P(B_i|A)$ as follows:

$$P(B_i|A) = \frac{P(B_i \cap A)}{P(A)} = \frac{P(B_i) \cdot P(A|B_i)}{P(A)}$$

where $P(A) = \sum_{i=1}^{k} P(B_i) \cdot P(A|B_i)$

Bayes′ rule

If $P(B_i)$ and $P(A|B_i)$ are given for all i, we can calculate $P(B_i|A)$ as follows:

$$P(B_i|A) = \frac{P(B_i \cap A)}{P(A)} = \frac{P(B_i) \cdot P(A|B_i)}{P(A)}$$

where $P(A) = \sum_{i=1}^{k} P(B_i) \cdot P(A|B_i)$

+ Examples

And now . . .

Book: Chapter 4.1

Throwing a die 6000 times . . .

Book: Chapter 4.1

Throwing a die 6000 times ...

The number of events will be approximately: $1000 \times$ '1', $1000 \times$ '2', $1000 \times$ '3', $1000 \times$ '4', $1000 \times$ '5', $1000 \times$ '6'

Book: Chapter 4.1

Throwing a die 6000 times . . .

The number of events will be approximately: $1000 \times$ '1', $1000 \times$ '2', $1000 \times$ '3', $1000 \times$ '4', $1000 \times$ '5', $1000 \times$ '6'
The total $\sharp$ eyes will be therefore approximately
$1000 \cdot 1 + 1000 \cdot 2 + 1000 \cdot 3 + 1000 \cdot 4 + 1000 \cdot 5 + 1000 \cdot 6 = 21 \cdot 10^3$

Book: Chapter 4.1

Throwing a die 6000 times ...

The number of events will be approximately: $1000 \times$ '1', $1000 \times$ '2', $1000 \times$ '3', $1000 \times$ '4', $1000 \times$ '5', $1000 \times$ '6'

The total $\sharp$ eyes will be therefore approximately

$1000 \cdot 1 + 1000 \cdot 2 + 1000 \cdot 3 + 1000 \cdot 4 + 1000 \cdot 5 + 1000 \cdot 6 = 21 \cdot 10^3$

The average $\sharp$ eyes is then approximately

Book: Chapter 4.1

Throwing a die 6000 times ...

The number of events will be approximately: $1000 \times$ '1', $1000 \times$ '2', $1000 \times$ '3', $1000 \times$ '4', $1000 \times$ '5', $1000 \times$ '6'

The total $\sharp$ eyes will be therefore approximately $1000 \cdot 1 + 1000 \cdot 2 + 1000 \cdot 3 + 1000 \cdot 4 + 1000 \cdot 5 + 1000 \cdot 6 = 21 \cdot 10^3$

The average $\sharp$ eyes is then approximately

$$\frac{10^3 \cdot 1 + 10^3 \cdot 2 + 10^3 \cdot 3 + 10^3 \cdot 4 + 10^3 \cdot 5 + 10^3 \cdot 6}{6 \cdot 10^3}$$

Book: Chapter 4.1

Throwing a die 6000 times ...

The number of events will be approximately: $1000 \times$ '1', $1000 \times$ '2', $1000 \times$ '3', $1000 \times$ '4', $1000 \times$ '5', $1000 \times$ '6'
The total $\sharp$ eyes will be therefore approximately
$1000 \cdot 1 + 1000 \cdot 2 + 1000 \cdot 3 + 1000 \cdot 4 + 1000 \cdot 5 + 1000 \cdot 6 = 21 \cdot 10^3$
The average $\sharp$ eyes is then approximately

$$\frac{10^3 \cdot 1 + 10^3 \cdot 2 + 10^3 \cdot 3 + 10^3 \cdot 4 + 10^3 \cdot 5 + 10^3 \cdot 6}{6 \cdot 10^3}$$
$$= \frac{1}{6} \cdot 1 + \frac{1}{6} \cdot 2 + \frac{1}{6} \cdot 3 + \frac{1}{6} \cdot 4 + \frac{1}{6} \cdot 5 + \frac{1}{6} \cdot 6$$

Book: Chapter 4.1

Throwing a die 6000 times ...

The number of events will be approximately: $1000 \times$ '1', $1000 \times$ '2', $1000 \times$ '3', $1000 \times$ '4', $1000 \times$ '5', $1000 \times$ '6'
The total $\sharp$ eyes will be therefore approximately
$1000 \cdot 1 + 1000 \cdot 2 + 1000 \cdot 3 + 1000 \cdot 4 + 1000 \cdot 5 + 1000 \cdot 6 = 21 \cdot 10^3$
The average $\sharp$ eyes is then approximately

$$\frac{10^3 \cdot 1 + 10^3 \cdot 2 + 10^3 \cdot 3 + 10^3 \cdot 4 + 10^3 \cdot 5 + 10^3 \cdot 6}{6 \cdot 10^3}$$
$$= \frac{1}{6} \cdot 1 + \frac{1}{6} \cdot 2 + \frac{1}{6} \cdot 3 + \frac{1}{6} \cdot 4 + \frac{1}{6} \cdot 5 + \frac{1}{6} \cdot 6 = 3\frac{1}{2}$$

Book: Chapter 4.1

Throwing a die 6000 times . . .

The number of events will be approximately: $1000 \times$ '1', $1000 \times$ '2', $1000 \times$ '3', $1000 \times$ '4', $1000 \times$ '5', $1000 \times$ '6'
The total $\sharp$ eyes will be therefore approximately
$1000\cdot 1+1000\cdot 2+1000\cdot 3+1000\cdot 4+1000\cdot 5+1000\cdot 6 = 21\cdot 10^3$
The average $\sharp$ eyes is then approximately

$$\frac{10^3\cdot 1+10^3\cdot 2+10^3\cdot 3+10^3\cdot 4+10^3\cdot 5+10^3\cdot 6}{6\cdot 10^3}$$
$$= \frac{1}{6}\cdot 1+\frac{1}{6}\cdot 2+\frac{1}{6}\cdot 3+\frac{1}{6}\cdot 4+\frac{1}{6}\cdot 5+\frac{1}{6}\cdot 6 = 3\frac{1}{2}$$
$$= P(\{1\})\cdot 1+P(\{2\})\cdot 2+P(\{3\})\cdot 3+P(\{4\})\cdot 4$$
$$+P(\{5\})\cdot 5+P(\{6\})\cdot 6$$

Book: Chapter 4.1

Throwing a die 6000 times ...

The number of events will be approximately: $1000 \times$ '1', $1000 \times$ '2', $1000 \times$ '3', $1000 \times$ '4', $1000 \times$ '5', $1000 \times$ '6'
The total $\sharp$ eyes will be therefore approximately
$1000 \cdot 1 + 1000 \cdot 2 + 1000 \cdot 3 + 1000 \cdot 4 + 1000 \cdot 5 + 1000 \cdot 6 = 21 \cdot 10^3$
The average $\sharp$ eyes is then approximately

$$\frac{10^3 \cdot 1 + 10^3 \cdot 2 + 10^3 \cdot 3 + 10^3 \cdot 4 + 10^3 \cdot 5 + 10^3 \cdot 6}{6 \cdot 10^3}$$
$$= \frac{1}{6} \cdot 1 + \frac{1}{6} \cdot 2 + \frac{1}{6} \cdot 3 + \frac{1}{6} \cdot 4 + \frac{1}{6} \cdot 5 + \frac{1}{6} \cdot 6 = 3\frac{1}{2}$$
$$= P(\{1\}) \cdot 1 + P(\{2\}) \cdot 2 + P(\{3\}) \cdot 3 + P(\{4\}) \cdot 4$$
$$+ P(\{5\}) \cdot 5 + P(\{6\}) \cdot 6$$
$$= \sum_{s \in S} s \cdot P(\{s\})$$

Expectation (Mean) of Discrete Random Variable

Expectation (mean, expected value) of discrete random variable X is defined as

$$E(X) = \mu_X = \sum_{x \in X} x \cdot f(x).$$

Expectation (Mean) of Discrete Random Variable

Expectation (mean, expected value) of discrete random variable X is defined as

$$E(X) = \mu_X = \sum_{x \in X} x \cdot f(x).$$

In the previous example: X : $\sharp$ eyes in a single throw.

Expectation (Mean) of Discrete Random Variable

Expectation (mean, expected value) of discrete random variable X is defined as

$$E(X) = \mu_X = \sum_{x \in X} x \cdot f(x).$$

In the previous example: X : $\sharp$ eyes in a single throw.

Example 2

Y : $\sharp$ eyes squared

Expectation (Mean) of Discrete Random Variable

Expectation (mean, expected value) of discrete random variable X is defined as

$$E(X) = \mu_X = \sum_{x \in X} x \cdot f(x).$$

In the previous example: X : $\sharp$ eyes in a single throw.

Example 2

Y : $\sharp$ eyes squared

$$E(Y) = \mu_Y = \sum_{y \in Y} y \cdot f(y)$$

Expectation (Mean) of Discrete Random Variable

Expectation (mean, expected value) of discrete random variable X is defined as

$$E(X) = \mu_X = \sum_{x \in X} x \cdot f(x).$$

In the previous example: X : $\sharp$ eyes in a single throw.

Example 2

Y : $\sharp$ eyes squared

$$\begin{aligned} E(Y) = \mu_Y = & \sum_{y \in Y} y \cdot f(y) \\ & = 1 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 9 \cdot \frac{1}{6} + 16 \cdot \frac{1}{6} + 25 \cdot \frac{1}{6} + 36 \cdot \frac{1}{6} \end{aligned}$$

Expectation (Mean) of Discrete Random Variable

Expectation (mean, expected value) of discrete random variable X is defined as

$$E(X) = \mu_X = \sum_{x \in X} x \cdot f(x).$$

In the previous example: X : $\sharp$ eyes in a single throw.

Example 2

Y : $\sharp$ eyes squared

$$\begin{aligned} E(Y) = \mu_Y &= \sum_{y \in Y} y \cdot f(y) \\ &= 1 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 9 \cdot \frac{1}{6} + 16 \cdot \frac{1}{6} + 25 \cdot \frac{1}{6} + 36 \cdot \frac{1}{6} \\ &= \frac{91}{6} \end{aligned}$$

Expectation (Mean) of Continuous Random Variable

Expectation (mean) of continuous random variable X is defined as

$$E(X) = \mu_X = \int_{-\infty}^{\infty} x \cdot f(x)\mathrm{d}x$$

Expectation (Mean) of Continuous Random Variable

Expectation (mean) of continuous random variable X is defined as

$$E(X) = \mu_X = \int_{-\infty}^{\infty} x \cdot f(x)\mathrm{d}x$$

Example

X is a continuous RV with PDF:

$$f(x) = \begin{cases} 8 - 2x, & 3 < x < 4, \\ 0, & \text{elsewhere} \end{cases}$$

What is $E(X)$?

Expectation (Mean) of Continuous Random Variable

Expectation (mean) of continuous random variable X is defined as

$$E(X) = \mu_X = \int_{-\infty}^{\infty} x \cdot f(x)\mathrm{d}x$$

Example

X is a continuous RV with PDF:

$$f(x) = \begin{cases} 8 - 2x, & 3 < x < 4, \\ 0, & \text{elsewhere} \end{cases}$$

What is $E(X)$?

$$E(X) = \mu_X = \int_{-\infty}^{\infty} x \cdot f(x)\mathrm{d}x$$

Expectation (Mean) of Continuous Random Variable

Expectation (mean) of continuous random variable X is defined as

$$E(X) = \mu_X = \int_{-\infty}^{\infty} x \cdot f(x)\mathrm{d}x$$

Example

X is a continuous RV with PDF:

$$f(x) = \begin{cases} 8 - 2x, & 3 < x < 4, \\ 0, & \text{elsewhere} \end{cases}$$

What is $E(X)$?

$$E(X) = \mu_X = \int_{-\infty}^{\infty} x \cdot f(x)\mathrm{d}x = \int_{3}^{4} x(8 - 2\,x)\mathrm{d}x$$

Expectation (Mean) of Continuous Random Variable

Expectation (mean) of continuous random variable X is defined as

$$E(X) = \mu_X = \int_{-\infty}^{\infty} x \cdot f(x) \mathrm{d}x$$

Example

X is a continuous RV with PDF:

$$f(x) = \begin{cases} 8 - 2x, & 3 < x < 4, \\ 0, & \text{elsewhere} \end{cases}$$

What is $E(X)$?

$$E(X) = \mu_X = \int_{-\infty}^{\infty} x \cdot f(x) \mathrm{d}x = \int_3^4 x(8 - 2\,x) \mathrm{d}x = \left[4\,x^2 - \frac{2}{3}x^3\right]_{x=3}^4$$

Expectation (Mean) of Continuous Random Variable

Expectation (mean) of continuous random variable X is defined as

$$E(X) = \mu_X = \int_{-\infty}^{\infty} x \cdot f(x)\mathrm{d}x$$

Example

X is a continuous RV with PDF:

$$f(x) = \begin{cases} 8-2x, & 3 < x < 4, \\ 0, & \text{elsewhere} \end{cases}$$

What is $E(X)$?

$$E(X) = \mu_X = \int_{-\infty}^{\infty} x \cdot f(x)\mathrm{d}x = \int_3^4 x(8-2\,x)\mathrm{d}x = \left[4\,x^2 - \frac{2}{3}x^3\right]_{x=3}^4$$

$$= 64 - \frac{128}{3} - (36-18) = \frac{64}{3} - 18 = 3\frac{1}{3}$$

And now . . .

Discrete RV

Let X be a discrete RV with PDF f and let g be an arbitrary real-valued function. Then the expectation (mean) of $g(X)$ is defined as follows:

Discrete RV

Let X be a discrete RV with PDF f and let g be an arbitrary real-valued function. Then the expectation (mean) of $g(X)$ is defined as follows:

$$E(g(X)) = \mu_{g(X)} = \sum_{x \in X} g(x)f(x)$$

Discrete RV

Let X be a discrete RV with PDF f and let g be an arbitrary real-valued function. Then the expectation (mean) of $g(X)$ is defined as follows:

$$E(g(X)) = \mu_{g(X)} = \sum_{x \in X} g(x)f(x)$$

Continuous RV

Let X be a continuous RV with PDF f and let g be an arbitrary real-valued function. Then the expectation of $g(X)$ is defined as follows:

Discrete RV

Let X be a discrete RV with PDF f and let g be an arbitrary real-valued function. Then the expectation (mean) of $g(X)$ is defined as follows:

$$E(g(X)) = \mu_{g(X)} = \sum_{x \in X} g(x) f(x)$$

Continuous RV

Let X be a continuous RV with PDF f and let g be an arbitrary real-valued function. Then the expectation of $g(X)$ is defined as follows:

$$E(g(X)) = \mu_{g(X)} = \int_{-\infty}^{\infty} g(x) f(x) \, \mathrm{d}x$$

Discrete RV

Let X be a discrete RV with PDF f and let g and h be arbitrary real-valued functions. Then the expectation (mean) of $g(X) + h(X)$ is defined as follows:

Discrete RV

Let X be a discrete RV with PDF f and let g and h be arbitrary real-valued functions. Then the expectation (mean) of $g(X) + h(X)$ is defined as follows:

$$\begin{aligned} E(g(X) + h(X)) &= \mu_{g(X)+h(X)} = \sum_x (g(x) + h(x)) f(x) \\ &= \sum_x g(x)\, f(x) + \sum_x h(x)\, f(x) = E(g(X)) + E(h(X)) \end{aligned}$$

Discrete RV

Let X be a discrete RV with PDF f and let g and h be arbitrary real-valued functions. Then the expectation (mean) of $g(X) + h(X)$ is defined as follows:

$$E(g(X) + h(X)) = \mu_{g(X)+h(X)} = \sum_{x} (g(x) + h(x)) f(x)$$
$$= \sum_{x} g(x)\, f(x) + \sum_{x} h(x)\, f(x) = E(g(X)) + E(h(X))$$

Continuous RV

Let X be a continuous RV with PDF f and let g and h be arbitrary real-valued functions. Then the expectation (mean) of $g(X) + h(X)$ is defined as follows:

Discrete RV

Let X be a discrete RV with PDF f and let g and h be arbitrary real-valued functions. Then the expectation (mean) of $g(X)+h(X)$ is defined as follows:

$$\begin{aligned} E(g(X)+h(X)) = \mu_{g(X)+h(X)} &= \sum_{x}(g(x)+h(x))f(x) \\ &= \sum_{x} g(x)\,f(x) + \sum_{x} h(x)\,f(x) = E(g(X))+E(h(X)) \end{aligned}$$

Continuous RV

Let X be a continuous RV with PDF f and let g and h be arbitrary real-valued functions. Then the expectation (mean) of $g(X)+h(X)$ is defined as follows:

$$\begin{aligned} E(g(X)+h(X)) = \mu_{g(X)+h(X)} &= \int_{-\infty}^{\infty}(g(x)+h(x))f(x)\,\mathrm{d}x \\ &= \int_{-\infty}^{\infty} g(x)\,f(x)\mathrm{d}x + \int_{-\infty}^{\infty} h(x)\,f(x)\mathrm{d}x = E(g(X))+E(h(X)) \end{aligned}$$

Other relations

- $E(aX + b) = aE(X) + b$

Other relations

- $E(aX + b) = aE(X) + b$
- $E(a\,X^2 + b\,X + c) = a\,E(X^2) + b\,E(X) + c$

Other relations

- $E(aX + b) = aE(X) + b$
- $E(aX^2 + bX + c) = aE(X^2) + bE(X) + c$
- $E(X + Y) = E(X) + E(Y)$

Other relations

- $E(aX + b) = aE(X) + b$
- $E(a\,X^2 + b\,X + c) = a\,E(X^2) + b\,E(X) + c$
- $E(X + Y) = E(X) + E(Y)$
- $E(g(X, Y) \pm h(X, Y)) = E(g(X, Y)) \pm E(h(X, Y))$

Example: Toss 2 dice

- X : $\sharp$ eyes on first die

Example: Toss 2 dice

- X : $\sharp$ eyes on first die
- Y : $\sharp$ eyes on second die

Example: Toss 2 dice

- X : $\sharp$ eyes on first die
- Y : $\sharp$ eyes on second die
- $g(X, Y)$: squared $\sharp$ eyes on first die: $g(x, y) = x^2$

Example: Toss 2 dice

- X : $\sharp$ eyes on first die
- Y : $\sharp$ eyes on second die
- $g(X, Y)$: squared $\sharp$ eyes on first die: $g(x, y) = x^2$
- $h(X, Y)$: product of $\sharp$ on the two dice: $h(x, y) = x\,y$

Example: Toss 2 dice

- X : $\sharp$ eyes on first die
- Y : $\sharp$ eyes on second die
- $g(X, Y)$: squared $\sharp$ eyes on first die: $g(x, y) = x^2$
- $h(X, Y)$: product of $\sharp$ on the two dice: $h(x, y) = x\,y$
- $E(g(X, Y) + h(X, Y)) = ?$

Example: Toss 2 dice

- X : $\sharp$ eyes on first die
- Y : $\sharp$ eyes on second die
- $g(X, Y)$: squared $\sharp$ eyes on first die: $g(x, y) = x^2$
- $h(X, Y)$: product of $\sharp$ on the two dice: $h(x, y) = x\,y$
- $E(g(X, Y) + h(X, Y)) = ?$

$$\begin{aligned} E(g(X, Y) + h(X, Y)) &= E(X^2 + X\,Y) = E(X^2) + E(XY) \\ &= \frac{91}{6} + E(XY) = ? \end{aligned}$$

Other relations - Independent RVs

Let X and Y be independent RVs. Then

$$E(XY) = E(X) \cdot E(Y)$$

Other relations - Independent RVs

Let X and Y be independent RVs. Then

$$E(XY) = E(X) \cdot E(Y)$$

Example: Toss of 2 dice

- X : $\sharp$ eyes on first die
- Y : $\sharp$ eyes on second die
- $g(X, Y)$: squared $\sharp$ eyes on first die: $g(x, y) = x^2$
- $h(X, Y)$: product of $\sharp$ on the two dice: $h(x, y) = x\,y$
- $E(g(X, Y) + h(X, Y)) = ?$

Other relations - Independent RVs

Let X and Y be independent RVs. Then

$$E(XY) = E(X) \cdot E(Y)$$

Example: Toss of 2 dice

- X : $\sharp$ eyes on first die
- Y : $\sharp$ eyes on second die
- $g(X, Y)$: squared $\sharp$ eyes on first die: $g(x, y) = x^2$
- $h(X, Y)$: product of $\sharp$ on the two dice: $h(x, y) = x\,y$
- $E(g(X, Y) + h(X, Y)) =?$
  X and Y independent. Hence

Other relations - Independent RVs

Let X and Y be independent RVs. Then

$$E(XY) = E(X) \cdot E(Y)$$

Example: Toss of 2 dice

- X : $\sharp$ eyes on first die
- Y : $\sharp$ eyes on second die
- $g(X, Y)$: squared $\sharp$ eyes on first die: $g(x, y) = x^2$
- $h(X, Y)$: product of $\sharp$ on the two dice: $h(x, y) = x\, y$
- $E(g(X, Y) + h(X, Y)) = ?$
  X and Y independent. Hence

  $$E(g(X, Y) + h(X, Y))$$

Other relations - Independent RVs

Let X and Y be independent RVs. Then

$$E(XY) = E(X) \cdot E(Y)$$

Example: Toss of 2 dice

- X : $\sharp$ eyes on first die
- Y : $\sharp$ eyes on second die
- $g(X, Y)$: squared $\sharp$ eyes on first die: $g(x, y) = x^2$
- $h(X, Y)$: product of $\sharp$ on the two dice: $h(x, y) = x\,y$
- $E(g(X, Y) + h(X, Y)) = ?$
  X and Y independent. Hence

$$E(g(X, Y) + h(X, Y)) = \frac{91}{6} + E(XY)$$

Other relations - Independent RVs

Let X and Y be independent RVs. Then

$$E(XY) = E(X) \cdot E(Y)$$

Example: Toss of 2 dice

- X : ♯ eyes on first die
- Y : ♯ eyes on second die
- $g(X, Y)$: squared ♯ eyes on first die: $g(x, y) = x^2$
- $h(X, Y)$: product of ♯ on the two dice: $h(x, y) = x\,y$
- $E(g(X, Y) + h(X, Y)) = ?$
  X and Y independent. Hence

$$\begin{aligned} E(g(X, Y) + h(X, Y)) &= \frac{91}{6} + E(XY) \\ &= \frac{91}{6} + E(X) \cdot E(Y) \end{aligned}$$

Other relations - Independent RVs

Let X and Y be independent RVs. Then

$$E(XY) = E(X) \cdot E(Y)$$

Example: Toss of 2 dice

- X : $\sharp$ eyes on first die
- Y : $\sharp$ eyes on second die
- $g(X, Y)$: squared $\sharp$ eyes on first die: $g(x, y) = x^2$
- $h(X, Y)$: product of $\sharp$ on the two dice: $h(x, y) = x\,y$
- $E(g(X, Y) + h(X, Y)) = ?$
  X and Y independent. Hence

$$\begin{aligned} E(g(X, Y) + h(X, Y)) &= \frac{91}{6} + E(XY) \\ &= \frac{91}{6} + E(X) \cdot E(Y) = \frac{91}{6} + \frac{7}{2} \cdot \frac{7}{2} \end{aligned}$$

Other relations - Independent RVs

Let X and Y be independent RVs. Then

$$E(XY) = E(X) \cdot E(Y)$$

Example: Toss of 2 dice

- X : $\sharp$ eyes on first die
- Y : $\sharp$ eyes on second die
- $g(X, Y)$: squared $\sharp$ eyes on first die: $g(x, y) = x^2$
- $h(X, Y)$: product of $\sharp$ on the two dice: $h(x, y) = x\, y$
- $E(g(X, Y) + h(X, Y)) = ?$
  X and Y independent. Hence

$$\begin{aligned} E(g(X, Y) + h(X, Y)) &= \frac{91}{6} + E(XY) \\ &= \frac{91}{6} + E(X) \cdot E(Y) = \frac{91}{6} + \frac{7}{2} \cdot \frac{7}{2} = \frac{329}{12} \end{aligned}$$

Exercise 4.17

x	-3	6	9
$f(x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Exercise 4.17

x	-3	6	9
$f(x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find

- $E(X)$
- $E(X^2)$
- $E\left\{(2X+1)^2\right\}$

Exercise 4.17

x	-3	6	9
$f(x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find

- $E(X)$
- $E(X^2)$
- $E\left\{(2X+1)^2\right\}$

Exercise 4.17

x	-3	6	9
$f(x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find

- $E(X)$
- $E(X^2)$
- $E\left\{(2X+1)^2\right\}$

- $E(X) = \frac{1}{6} \cdot (-3) + \frac{1}{2} \cdot 6 + \frac{1}{3} \cdot 9 = 5\frac{1}{2}$

Exercise 4.17

x	-3	6	9
$f(x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find

- $E(X)$
- $E(X^2)$
- $E\left\{(2X+1)^2\right\}$

- $E(X) = \frac{1}{6} \cdot (-3) + \frac{1}{2} \cdot 6 + \frac{1}{3} \cdot 9 = 5\frac{1}{2}$
- $E(X^2) = \frac{1}{6} \cdot (-3)^2 + \frac{1}{2} \cdot 6^2 + \frac{1}{3} \cdot 9^2 = 46\frac{1}{2}$

Exercise 4.17

x	-3	6	9
$f(x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find

- $E(X)$
- $E(X^2)$
- $E\left\{(2X+1)^2\right\}$

- $E(X) = \frac{1}{6}\cdot(-3) + \frac{1}{2}\cdot 6 + \frac{1}{3}\cdot 9 = 5\frac{1}{2}$
- $E(X^2) = \frac{1}{6}\cdot(-3)^2 + \frac{1}{2}\cdot 6^2 + \frac{1}{3}\cdot 9^2 = 46\frac{1}{2}$
- $E\left\{(2X+1)^2\right\} = E(4\,X^2 + 4\,X + 1) =$
  $4\,E(X^2) + 4\,E(X) + 1 = 4\cdot 46\frac{1}{2} + 4\cdot 5\frac{1}{2} + 1 = 209$

And now . . .

Try all odd exercises from pages 117-118.